SİVAS UNIVERSITY OF SCIENCE AND TECHNOLOGY

SİVAS UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF ENGINEERING AND NATURAL SCIENCES

EEE 212 - INTRODUCTION TO COMMUNICATION SYSTEMS LABORATORY

Experiments Manual Report

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Experiment 1: Fourier Transform and Time-Frequency Domain Analysis of Signals

1. Objective of the Experiment

The objective of this experiment is to introduce students to the time-domain and frequency-domain representations of continuous-time signals using MATLAB. Students will generate basic communication signals, visualize their time-domain waveforms, and obtain their frequency spectra using the Fast Fourier Transform (FFT).

The experiment focuses on a sinusoidal signal, an exponentially damped cosine signal, and a rectangular pulse. By comparing theoretical Fourier-transform properties with numerically obtained spectra, students will develop an understanding of frequency components, spectral broadening, frequency resolution, and the relationship between signal duration and bandwidth.

At the end of the experiment, students are expected to:

- Generate basic continuous-time signal models in MATLAB.
- Apply the FFT to uniformly sampled signals.
- Construct and interpret a centered two-sided frequency axis.
- Identify the positive- and negative-frequency components of sinusoidal signals.
- Explain the effect of exponential damping on spectral shape.
- Examine the inverse relationship between rectangular-pulse width and bandwidth.

2. Theoretical Background

2.1. Time- and Frequency-Domain Representations

A signal may be represented either as a function of time or as a function of frequency. The time-domain representation describes how the amplitude of a signal varies with time, whereas the frequency-domain representation shows the frequency components contained in the signal.

For a continuous-time signal x(t), the Fourier transform is defined as

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\, dt$$

where x(t) is the time-domain signal, X(f) is the frequency-domain representation, and f is frequency in hertz. The inverse Fourier transform is

$$x(t) = \int_{-\infty}^{+\infty} X(f)e^{j2\pi ft}\, df$$

The Fourier transform decomposes a signal into sinusoidal components with different frequencies, amplitudes, and phases.

2.2. Fourier Transform of a Cosine Signal

Consider the sinusoidal signal

$$x(t) = \cos(2\pi f_0 t)$$

Using Euler's identity,

$$\cos(2\pi f_0 t) = \tfrac{1}{2}e^{j2\pi f_0 t} + \tfrac{1}{2}e^{-j2\pi f_0 t}$$

its Fourier transform is

$$X(f) = ½\delta(f - f_0) + ½\delta(f + f_0)$$

Therefore, a real cosine signal with frequency f_0 produces two symmetric components at $f = +f_0$ and $f = -f_0$ in the two-sided magnitude spectrum.

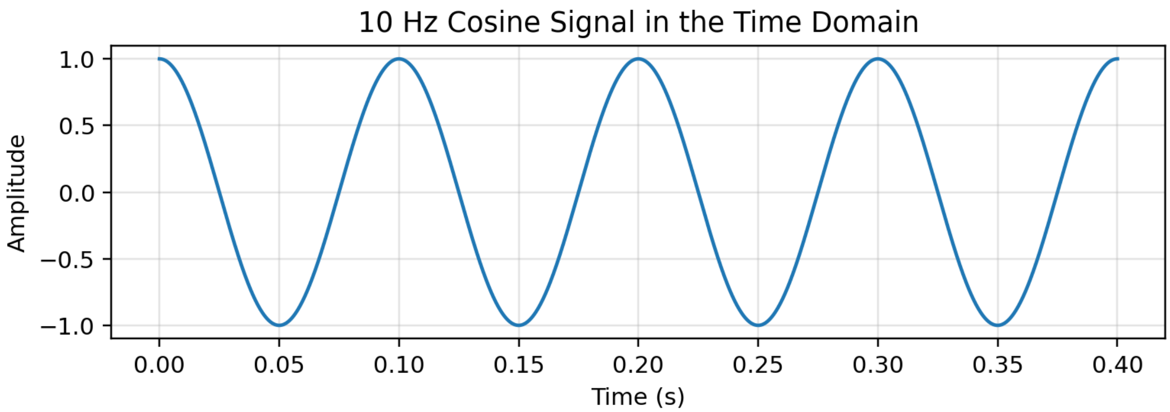

Fig. 1.1. Time-domain representation of a 10 Hz cosine signal.

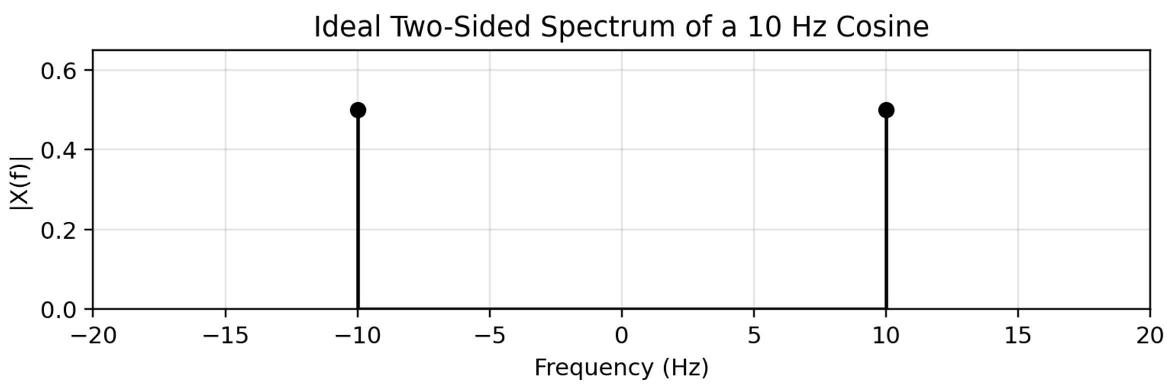

Fig. 1.2. Ideal two-sided magnitude spectrum of a 10 Hz cosine signal.

2.3. Exponentially Damped Cosine Signal

An exponentially damped cosine signal may be expressed as

$$x(t) = e^{-at}\cos(2\pi f_0 t)u(t), \quad a > 0$$

where a is the damping coefficient, f_0 is the oscillation frequency, and u(t) is the unit-step function. Unlike an ideal cosine extending over all time, the amplitude of a damped cosine decreases with time. Its finite effective duration causes the frequency content to spread around $+f_0$ and $-f_0$.

$$X(f) = ½[1/(a + j2\pi(f - f_0)) + 1/(a + j2\pi(f + f_0))]$$

A larger damping coefficient causes faster decay in the time domain and a broader spectrum in the frequency domain.

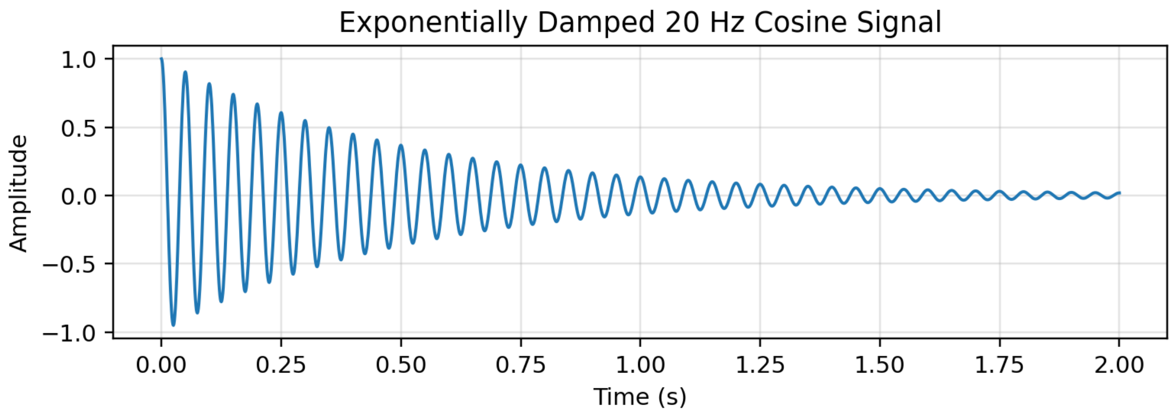

Fig. 1.3. Time-domain waveform of an exponentially damped 20 Hz cosine signal.

2.4. Rectangular Pulse Signal

A unit-amplitude rectangular pulse with width T can be defined as

$$x(t) = 1,\ |t| \le T/2; \qquad x(t) = 0,\ |t| > T/2$$

or equivalently as x(t) = rect(t/T). Its Fourier transform is

$$X(f) = T\,\mathrm{sinc}(fT)$$

where the normalized sinc function is sinc(x) = sin(πx)/(πx). The first spectral zeros occur at f = ±1/T. Therefore, the null-to-null width of the main lobe is

$$BW_{\text{null-to-null}} = 2/T$$

This result demonstrates the inverse relationship between pulse width and bandwidth: a wider pulse produces a narrower spectrum, whereas a narrower pulse produces a wider spectrum.

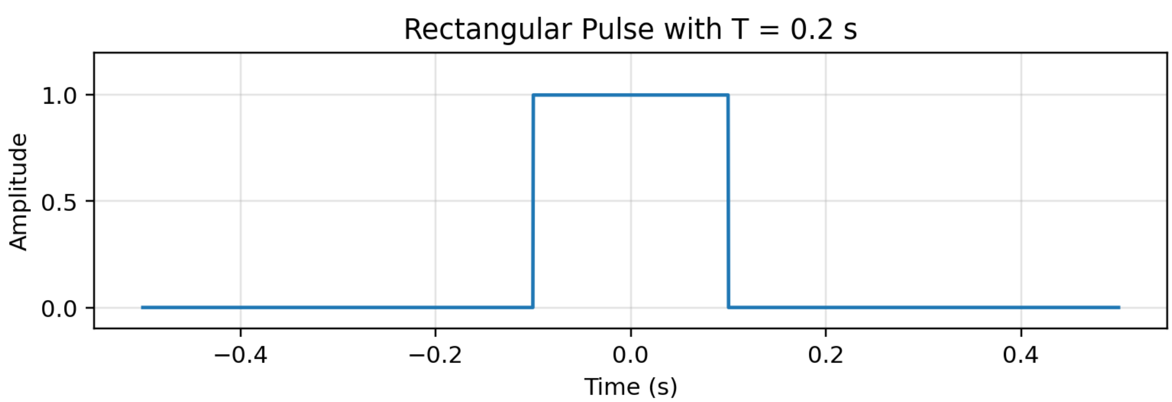

Fig. 1.4. Rectangular pulse with a duration of T = 0.2 s.

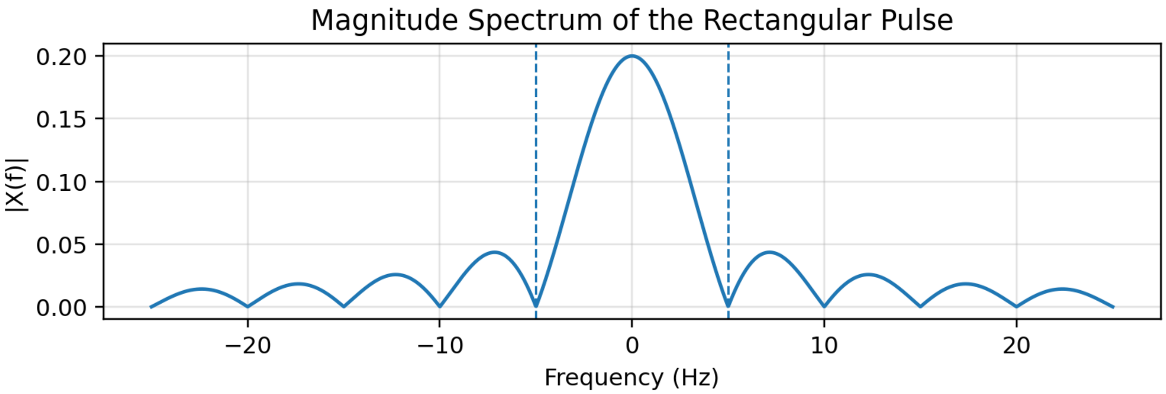

Fig. 1.5. Sinc-shaped magnitude spectrum of the rectangular pulse.

2.5. Fast Fourier Transform

MATLAB calculates the discrete Fourier transform efficiently using the fft command. For a sampled sequence containing N samples,

$$X[k] = \Sigma_{n=0}{}^{N-1}\, x[n]e^{-j2}\pi^{kn}/^{N}$$

The frequency resolution is

$$\Delta f = F_s/N$$

where F_s is the sampling frequency. For a centered two-sided spectrum, the frequency axis may be constructed using

$$f_k = (-N/2, \ldots, N/2 - 1)(F_s/N)$$

The fftshift command moves the zero-frequency component to the center of the spectrum. To obtain a properly scaled magnitude spectrum, the FFT result should be normalized by N.

3. Materials Used

- Computer workstation
- MATLAB software and MATLAB Script Editor
- Introduction to Communication Systems course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are: cos, exp, abs, fft, fftshift, length, plot, subplot, grid, xlabel, ylabel, and title.

4. Preliminary (PreLab)

The following tasks must be completed before attending the laboratory. All theoretical sketches and calculations must be included in the pre-laboratory submission.

4.1. Sinusoidal Signal

Consider $x_1(t) = \cos(2\pi 10t)$.

1. Determine the fundamental frequency of the signal.
2. Determine the positive- and negative-frequency components expected in the two-sided spectrum.

3. Sketch the theoretical time-domain waveform.
4. Sketch the theoretical magnitude spectrum.
5. Explain why a real cosine signal contains two symmetric spectral components.

4.2. Damped Cosine Signal

Consider $x_2(t) = e^{-2t} \cos(2\pi 20t)u(t)$.

1. Determine the oscillation frequency.
2. Determine the damping coefficient.
3. Sketch the expected time-domain waveform.
4. State the frequencies around which the spectrum is expected to be concentrated.
5. Explain whether the spectrum is expected to be narrower or wider than that of an ideal cosine signal.

4.3. Rectangular Pulse

Consider $x_3(t) = \mathrm{rect}(t/T)$, where $T = 0.2$ s.

1. Calculate the first positive and negative zero frequencies using $f_{zero} = \pm 1/T$.
2. Calculate the theoretical null-to-null bandwidth using $BW = 2/T$.
3. Sketch the time-domain rectangular pulse.
4. Sketch the expected sinc-shaped frequency spectrum.

Table 1.1. Preliminary calculations.

Signal	Parameter	Theoretical Value
Cosine signal	Fundamental frequency, f_0	
Cosine signal	Positive spectral component	
Cosine signal	Negative spectral component	
Damped cosine	Oscillation frequency	
Damped cosine	Damping coefficient	
Rectangular pulse	First positive zero	
Rectangular pulse	First negative zero	
Rectangular pulse	Null-to-null bandwidth	

5. Procedure

Part 1: Analysis of a Sinusoidal Signal

Consider $x_1(t) = \cos(2\pi f_0 t)$, where $f_0 = 10$ Hz.

1. Open MATLAB and create a new script file.
2. Set the sampling frequency to $F_s = 1000$ Hz.

3. Define a one-second time vector using the code below.

```
Fs = 1000;
t = 0:1/Fs:1-1/Fs;
```

4. Generate the cosine signal using $f_0 = 10$ Hz.
5. Plot the signal in the time domain.
6. Determine the number of samples using the length command.
7. Calculate the FFT of the signal and normalize it by the total number of samples.
8. Apply fftshift to center the spectrum around zero frequency.
9. Construct the two-sided frequency axis.
10. Display the time-domain signal and its magnitude spectrum in the same figure using subplot(2,1,...).
11. Determine the frequencies at which the two major peaks occur.
12. Compare the measured peak frequencies with the theoretical values.

Part 2: Analysis of a Damped Cosine Signal

Consider $x_2(t) = e^{-2t} \cos(2\pi 20t)$, $t \geq 0$.

1. Use a sampling frequency of $F_s = 1000$ Hz.
2. Define the observation interval as $0 \leq t < 2$ s.
3. Generate the exponentially damped cosine signal.
4. Plot the time-domain waveform.
5. Calculate and normalize its FFT.
6. Apply fftshift and construct the centered frequency axis.
7. Display the time-domain signal and frequency spectrum in the same figure.
8. Identify the frequencies around which the spectral peaks occur.
9. Compare the spectrum with the line spectrum of the ideal cosine signal.
10. Comment on the effect of exponential damping on spectral width.

Part 3: Analysis of a Rectangular Pulse

Consider a unit-amplitude rectangular pulse with $T = 0.2$ s.

1. Use a sampling frequency of $F_s = 1000$ Hz.
2. Define the time interval as $-1 \leq t < 1$ s.
3. Generate the rectangular pulse using the logical expression below.

```
x3 = double(abs(t) <= T/2);
```

4. Plot the rectangular pulse in the time domain.
5. Calculate and normalize its FFT.
6. Apply fftshift and construct the centered frequency axis.
7. Display the time-domain pulse and frequency spectrum in the same figure.
8. Determine the first positive and negative zero frequencies from the spectrum.
9. Calculate the measured null-to-null bandwidth and compare it with the theoretical value.
10. Repeat the analysis for $T = 0.1$ s and $T = 0.4$ s.
11. Observe how the spectrum changes as the pulse width is increased or decreased.

6. Evaluation of Results

Tabulate the data obtained during the experiment and compare the theoretical and MATLAB results.

Table 1.2. Frequency components of the sinusoidal signal.

Quantity	Theoretical Value	MATLAB Result	Percentage Error
Positive-frequency peak			
Negative-frequency peak			

The percentage error must be calculated using

$$\% \text{ Error} = |(\text{Measured Value} - \text{Theoretical Value})/\text{Theoretical Value}| \times 100$$

Table 1.3. Analysis of the damped cosine signal.

Parameter	Theoretical Value	MATLAB Result
Oscillation frequency		
Positive spectral center		
Negative spectral center		
Approximate spectral width		
Maximum time-domain amplitude		

Table 1.4. Pulse width and bandwidth comparison.

Pulse Width T	Theoretical First Zero 1/T	Measured First Zero	Theoretical BW 2/T	Measured BW
0.1 s				
0.2 s				
0.4 s				

Questions

1. Why are two symmetric frequency components observed for a real cosine signal?
2. At which frequencies do the spectral peaks of the 10 Hz cosine signal occur?
3. Why does the FFT spectrum not consist of mathematically ideal impulse functions?
4. How does finite observation time affect the obtained spectrum?
5. How does exponential damping affect the effective time duration of the signal?
6. Why does the damped cosine exhibit a broader spectrum than the ideal cosine?
7. What is the relationship between rectangular-pulse width and frequency-domain bandwidth?
8. What happens to the spectrum when the pulse width is reduced from 0.2 s to 0.1 s?
9. What happens to the spectrum when the pulse width is increased from 0.2 s to 0.4 s?
10. How does increasing the number of samples affect the frequency resolution?

11. Compare the theoretical and MATLAB-obtained results and discuss possible differences.

Submission Requirements

- MATLAB script file with the .m extension
- Figures containing the time- and frequency-domain plots
- Completed result tables
- A brief interpretation of each signal spectrum

Experiment 2: Verification of Fourier Transform Properties Using MATLAB

1. Objective of the Experiment

The objective of this experiment is to verify the fundamental properties of the Fourier transform by using MATLAB. The experiment focuses on linearity, time shifting, time scaling, frequency shifting, the convolution theorem, and Parseval's energy relation. Students will generate representative signals in the time domain, compute their spectra using the Fast Fourier Transform (FFT), and compare the numerical results with the corresponding theoretical expressions. Through these applications, students will observe how operations performed on a signal in the time domain modify its magnitude and phase spectra and will develop the ability to interpret Fourier-transform properties in communication-system analysis.

At the end of the experiment, students are expected to:

- Verify the linearity property of the Fourier transform numerically.
- Explain the effects of time shifting on magnitude and phase spectra.
- Observe the inverse relationship between time scaling and spectral width.
- Demonstrate frequency translation by multiplying a baseband signal by a carrier.
- Verify the convolution theorem using time- and frequency-domain calculations.
- Compare signal energy calculated in the time and frequency domains using Parseval's relation.

2. Theoretical Background

2.1. Fourier Transform Pair

For a continuous-time signal x(t), the Fourier transform and inverse Fourier transform are defined as

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\, dt$$
$$x(t) = \int_{-\infty}^{+\infty} X(f)e^{j2\pi ft}\, df$$

The Fourier transform expresses a signal as a combination of frequency components. Its properties allow many time-domain operations to be interpreted directly in the frequency domain.

2.2. Linearity

If $x_1(t) \leftrightarrow X_1(f)$ and $x_2(t) \leftrightarrow X_2(f)$, then the Fourier transform of a weighted sum is

$$a\, x_1(t) + b\, x_2(t) \ \leftrightarrow\ aX_1(f) + bX_2(f)$$

Therefore, each signal component may be transformed separately and the results may subsequently be combined using the same coefficients.

2.3. Time Shifting

A delay of t_0 seconds in the time domain produces a frequency-dependent phase factor in the frequency domain:

$$x(t - t_0) \ \leftrightarrow\ X(f)e^{-j2\pi ft0}$$

Time shifting does not change the magnitude spectrum. It changes only the phase spectrum by adding the linear phase term $-2\pi ft_0$.

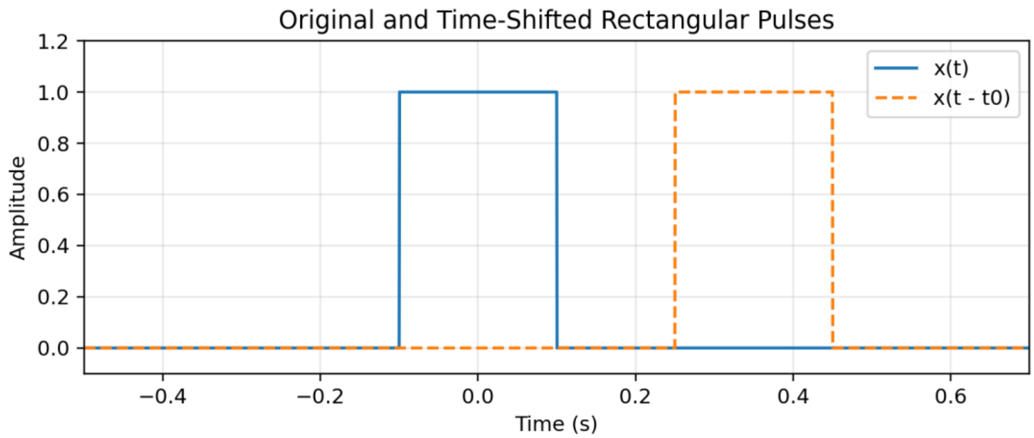

Fig. 2.1. Original and time-shifted rectangular pulses.

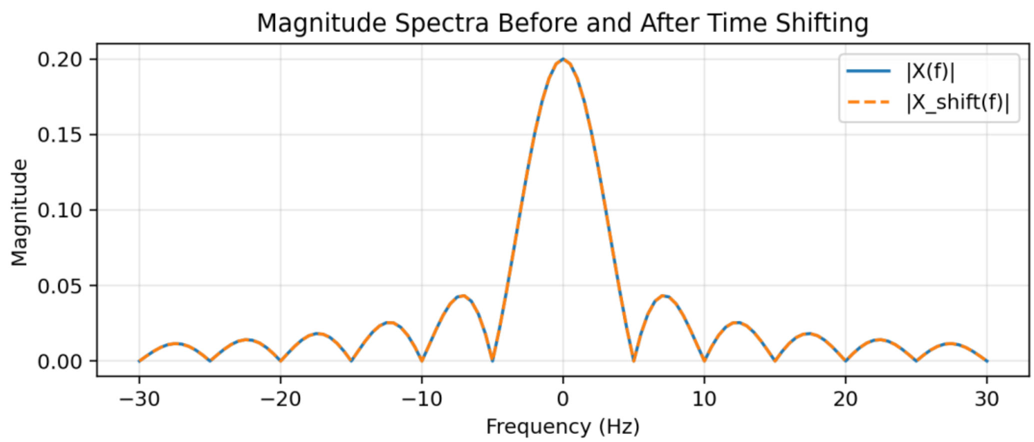

Fig. 2.2. Magnitude spectra of the original and shifted pulses.

2.4. Time Scaling

If a signal is compressed or expanded in time, its Fourier transform is modified according to

$$x(at) \leftrightarrow (1/|a|)X(f/a)$$

For $|a| > 1$, the signal is compressed in time and expanded in frequency. For $0 < |a| < 1$, the signal is expanded in time and compressed in frequency. The amplitude factor $1/|a|$ preserves the appropriate area and energy relationships.

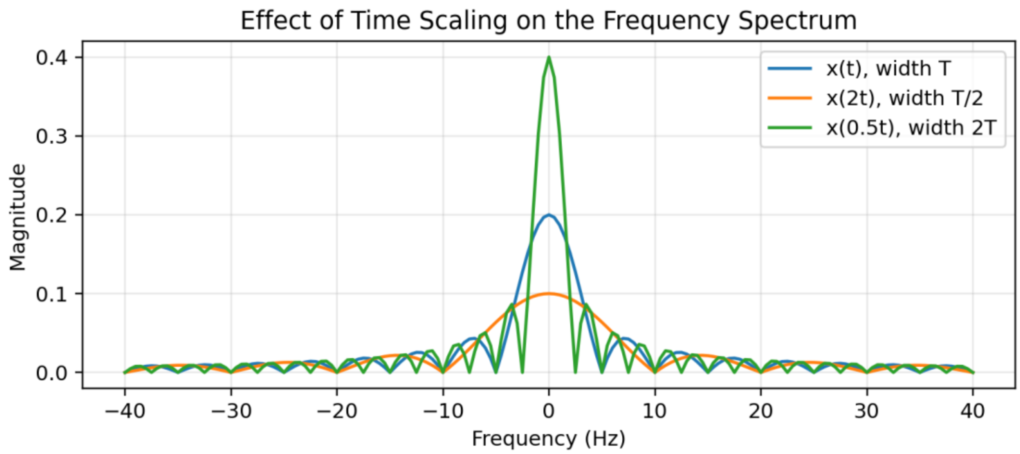

Fig. 2.3. Effect of time scaling on the spectrum of a rectangular pulse.

2.5. Frequency Shifting and Modulation

Multiplication of a signal by a complex exponential shifts its spectrum in frequency:

$$x(t)e^{j2}\pi^{fct} \leftrightarrow X(f - f_c)$$

For a real cosine carrier, the modulation property becomes

$$x(t)\cos(2\pi f_c t) \leftrightarrow \tfrac{1}{2}[X(f - f_c) + X(f + f_c)]$$

Thus, multiplication by a cosine generates two shifted copies of the baseband spectrum centered at +f_c and −f_c. This property forms the mathematical basis of amplitude modulation.

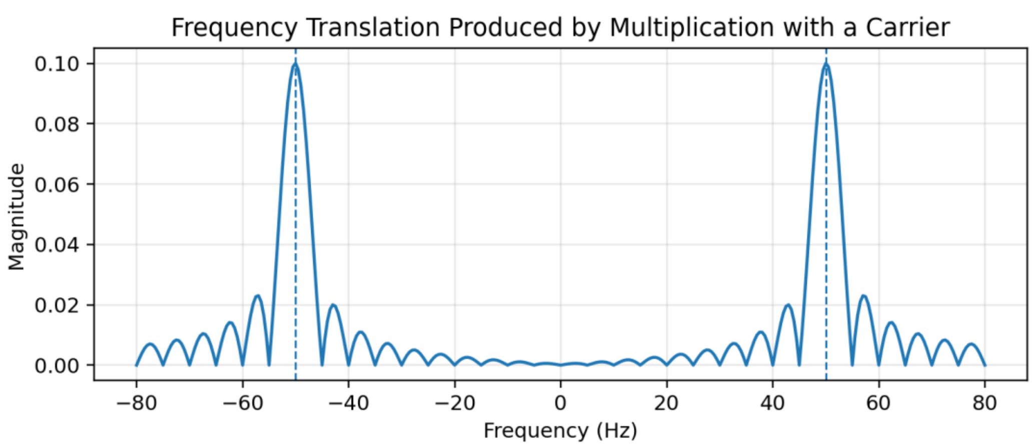

Fig. 2.4. Frequency translation of a rectangular baseband signal.

2.6. Convolution Theorem

The convolution of two signals in the time domain corresponds to multiplication of their Fourier transforms:

$$x_1(t) * x_2(t) \leftrightarrow X_1(f)X_2(f)$$

Conversely, multiplication of two signals in time corresponds to convolution of their spectra. The convolution theorem is widely used in the analysis of filters and linear time-invariant systems.

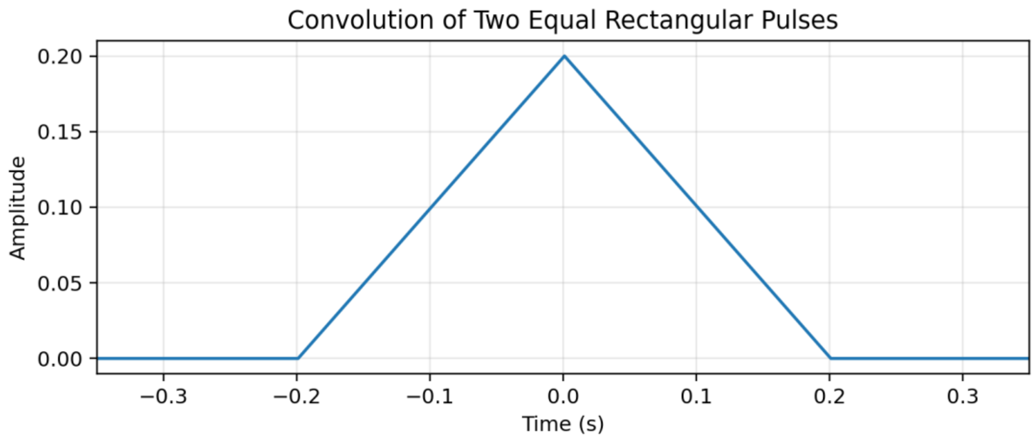

Fig. 2.5. Convolution of two equal rectangular pulses.

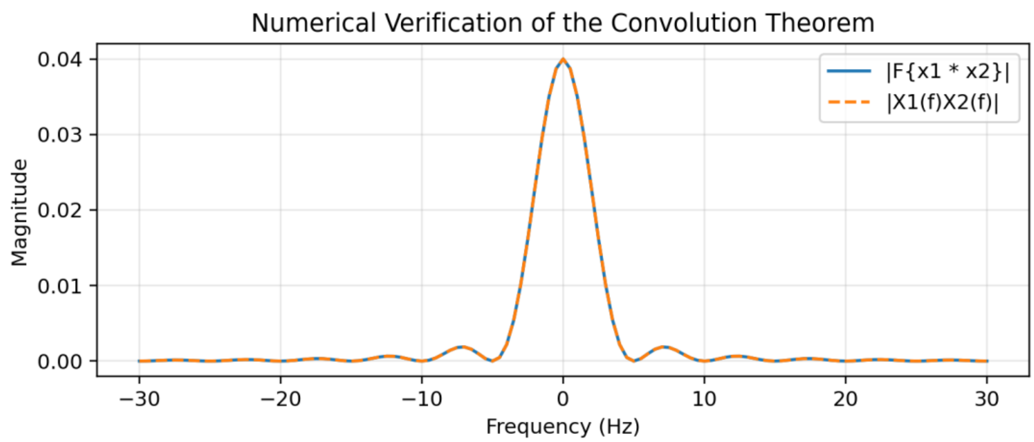

Fig. 2.6. Comparison of the Fourier transform of the convolution and the product $X_1(f)X_2(f)$.

2.7. Parseval's Energy Relation

For the Fourier-transform convention used in this laboratory, the total signal energy may be calculated equivalently in the time or frequency domain:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$$

In numerical calculations, the integrals are approximated by sums using the time interval Δt and the frequency interval Δf.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Script Editor
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- fft, fftshift, ifftshift and fftfreq-equivalent frequency-vector construction
- conv and trapz

- abs, angle and unwrap
- plot, subplot, legend, grid, xlabel, ylabel and title

4. Preliminary (PreLab)

Complete the following tasks before attending the laboratory. All theoretical sketches and calculations must be included in the pre-laboratory submission.

4.1. Linearity and Time Shifting

$$x_1(t) = \mathrm{rect}(t/0.2)$$
$$x_2(t) = \mathrm{rect}((t - 0.35)/0.2)$$
$$y(t) = 1.5x_1(t) - 0.8x_2(t)$$

1. Write Y(f) in terms of $X_1(f)$ and $X_2(f)$ by using linearity.
2. Write the Fourier transform of $x_2(t)$ in terms of the transform of $x_1(t)$.
3. State whether the magnitude spectra of $x_1(t)$ and $x_2(t)$ are expected to be identical.
4. Determine the phase term introduced by the delay $t_0 = 0.35$ s.

4.2. Time Scaling and Frequency Translation

$$x(t) = \mathrm{rect}(t/0.2)$$

1. Determine the pulse widths of x(2t) and x(0.5t).
2. Calculate the first positive spectral-zero frequency for x(t), x(2t), and x(0.5t).
3. For $s(t) = x(t)\cos(2\pi 50t)$, determine the center frequencies of the two shifted spectral copies.
4. Sketch the expected spectrum of s(t) and mark the carrier frequency and the first spectral zeros.

4.3. Convolution and Energy

1. Sketch the convolution of two equal rectangular pulses, each having width T = 0.2 s and unit amplitude.
2. Determine the total duration and maximum amplitude of the resulting triangular signal.
3. Calculate the theoretical energy of x(t) = rect(t/0.2).
4. State the relationship expected between the time-domain and frequency-domain energy values.

Table 2.1. Preliminary calculations for Fourier-transform properties.

Property	Quantity	Theoretical result
Time shifting	Delay t_0	
Time shifting	Additional phase term	
Time scaling	Width of x(2t)	
Time scaling	Width of x(0.5t)	
Frequency shifting	Positive spectral center	
Frequency shifting	Negative spectral center	
Convolution	Output duration	
Parseval	Theoretical signal energy	

5. Procedure

Part 1: Verification of Linearity and Time Shifting

1. Open MATLAB and create a new script file.
2. Set the sampling frequency to F_s = 2000 Hz and construct the time interval $-1 \leq t < 1$ s.
3. Generate $x_1(t)$ = rect(t/0.2) and $x_2(t)$ = rect((t − 0.35)/0.2) using logical expressions.
4. Construct y(t) = $1.5x_1(t) - 0.8x_2(t)$.
5. Calculate the continuous-time Fourier-transform approximations $X_1(f)$, $X_2(f)$, and Y(f) using the FFT and the sampling interval Δt.
6. Calculate Y_L(f) = $1.5X_1(f) - 0.8X_2(f)$.
7. Plot |Y(f)| and |Y_L(f)| on the same axes and calculate the maximum absolute difference.
8. Plot the magnitude spectra of $x_1(t)$ and $x_2(t)$ and verify that they overlap.
9. Plot and compare the unwrapped phase spectra in the frequency range where the magnitude is sufficiently large.
10. Determine the phase slope introduced by the delay and compare it with $-2\pi t_0$.

```
Fs = 2000;
dt = 1/Fs;
t = -1:dt:1-dt;
T = 0.2;  t0 = 0.35;
x1 = double(abs(t) <= T/2);
x2 = double(abs(t-t0) <= T/2);
y  = 1.5*x1 - 0.8*x2;
```

Part 2: Verification of Time Scaling and Frequency Shifting

1. Using the same sampling parameters, generate x(t) = rect(t/0.2).
2. Generate the compressed signal x(2t) and the expanded signal x(0.5t).
3. Calculate and plot the magnitude spectra of the three signals on the same frequency axis.
4. Determine the first positive spectral zero for each signal.
5. Compare the measured values with those predicted by the time-scaling property.
6. Set f_c = 50 Hz and generate s(t) = x(t)cos(2πf_ct).
7. Plot x(t), the carrier, and s(t) in the time domain.
8. Calculate the spectrum of s(t) and identify the shifted copies centered at +50 Hz and −50 Hz.
9. Measure the first spectral zeros around each center frequency and compare them with the baseband bandwidth.

```
x_fast = double(abs(2*t) <= T/2);      % x(2t)
x_slow = double(abs(0.5*t) <= T/2);    % x(0.5t)
fc = 50;
s = x1 .* cos(2*pi*fc*t);
```

Part 3: Verification of the Convolution Theorem and Parseval's Relation

1. Generate two unit-amplitude rectangular pulses $x_1(t)$ and $x_2(t)$, each with width T = 0.2 s.
2. Calculate their linear convolution by using the conv command and multiply the result by Δt.
3. Construct the correct time vector for the full convolution result.
4. Plot the convolution result and verify that it has a triangular shape.
5. Calculate the Fourier transforms $X_1(f)$ and $X_2(f)$.
6. Calculate the Fourier transform of the convolution result and compare it with $X_1(f)X_2(f)$.
7. Normalize the comparison consistently and calculate the maximum relative difference.
8. Calculate the energy of $x_1(t)$ in the time domain using $\Sigma|x_1[n]|^2\Delta t$.
9. Calculate the energy in the frequency domain using $\Sigma|X_1[k]|^2\Delta f$.
10. Calculate the percentage difference between the two energy values.

```
yconv = conv(x1,x1,'full') * dt;
ty = (2*t(1)):dt:(2*t(end));
X1 = fftshift(fft(ifftshift(x1))) * dt;
df = Fs/length(t);
E_time = sum(abs(x1).^2) * dt;
E_freq = sum(abs(X1).^2) * df;
```

6. Evaluation of Results

Tabulate the theoretical and MATLAB-obtained results. Percentage error is calculated as

$$\%\,\text{Error} = |(\text{MATLAB value} - \text{Theoretical value})/(\text{Theoretical value})| \times 100$$

Table 2.2. Verification of linearity and time shifting.

Quantity	Theoretical value	MATLAB value	Percentage error
Maximum difference between Y(f) and 1.5X_1(f) − 0.8X_2(f)			
Magnitude-spectrum difference between x_1(t) and x_2(t)			
Measured phase slope			
Delay calculated from phase slope			

Table 2.3. Time-scaling results.

Signal	Pulse width	Theoretical first zero	Measured first zero	Null-to-null bandwidth
x(t)				
x(2t)				
x(0.5t)				

Table 2.4. Frequency-translation results.

Quantity	Theoretical value	MATLAB value
Positive spectral center		
Negative spectral center		
Lower zero around +f_c		
Upper zero around +f_c		
Bandwidth of each shifted copy		

Table 2.5. Convolution and Parseval results.

Quantity	Theoretical value	MATLAB value	Percentage difference
Convolution duration			
Peak convolution amplitude			
Spectral-domain mismatch			

Quantity	Theoretical value	MATLAB value	Percentage difference
Time-domain energy			
Frequency-domain energy			

Questions

1. Does the Fourier transform of the weighted sum agree with the weighted sum of the individual Fourier transforms? Explain any small numerical difference.
2. Why does a time shift leave the magnitude spectrum unchanged?
3. How can the amount of time delay be estimated from the slope of the phase spectrum?
4. What happens to spectral width when a signal is compressed in time?
5. Why does multiplication by a cosine create two shifted spectral copies?
6. At which frequencies are the shifted spectra centered in this experiment?
7. Why is the convolution of two equal rectangular pulses triangular?
8. How closely does the Fourier transform of the convolution match the product $X_1(f)X_2(f)$?
9. Why must Δt and Δf be included in numerical approximations of continuous-time transforms and energies?
10. Compare the time-domain and frequency-domain energy values and discuss the sources of any discrepancy.

Experiment 3: Frequency Translation and Amplitude Modulation

1. Objective of the Experiment

The objective of this experiment is to examine frequency translation and the basic forms of amplitude modulation using MATLAB. Students will first generate a low-frequency message signal and a higher-frequency carrier signal, then observe how multiplication transfers the baseband frequency components to frequencies around the carrier. The experiment also investigates conventional amplitude modulation (AM) and double-sideband suppressed-carrier (DSB-SC) modulation in both the time and frequency domains. By comparing theoretical spectral components with FFT results, students will identify carrier and sideband frequencies, evaluate the effect of the modulation index, and distinguish between transmitted-carrier and suppressed-carrier modulation.

At the end of the experiment, students are expected to:

- Generate message, carrier, conventional AM, and DSB-SC signals in MATLAB.
- Explain frequency translation through multiplication by a sinusoidal carrier.
- Identify the carrier, upper-sideband, and lower-sideband components in an AM spectrum.
- Calculate the bandwidth of single-tone AM and DSB-SC signals.
- Relate the modulation index to the AM envelope and sideband amplitudes.
- Explain why the carrier component is absent from a DSB-SC spectrum.
- Compare conventional AM and DSB-SC signals in terms of spectral content and transmitted power.

2. Theoretical Background

2.1. Baseband and Carrier Signals

A communication system transfers information from a source to a destination through a channel. The information-bearing signal is generally referred to as the message or baseband signal. Because a baseband signal is usually concentrated at relatively low frequencies, it is shifted to a higher-frequency region before transmission. This operation is called modulation, and the high-frequency sinusoid used for the translation is called the carrier signal.

$$m(t) = A_m \cos(2\pi f_m t)$$

$$c(t) = A_c \cos(2\pi f_c t), \quad f_c \gg f_m$$

2.2. Signal Multiplication and Frequency Translation

For the single-tone signals

$$m(t) = \cos(2\pi f_m t), \quad c(t) = \cos(2\pi f_c t)$$

their product is obtained by using the product-to-sum identity:

$$m(t)c(t) = \tfrac{1}{2}\cos[2\pi(f_c - f_m)t] + \tfrac{1}{2}\cos[2\pi(f_c + f_m)t]$$

Therefore, multiplication produces two components at the difference and sum frequencies. In the frequency domain, the baseband spectrum is shifted to the regions centered at $+f_c$ and $-f_c$:

$$m(t)\cos(2\pi f_c t) \leftrightarrow \tfrac{1}{2}[M(f - f_c) + M(f + f_c)]$$

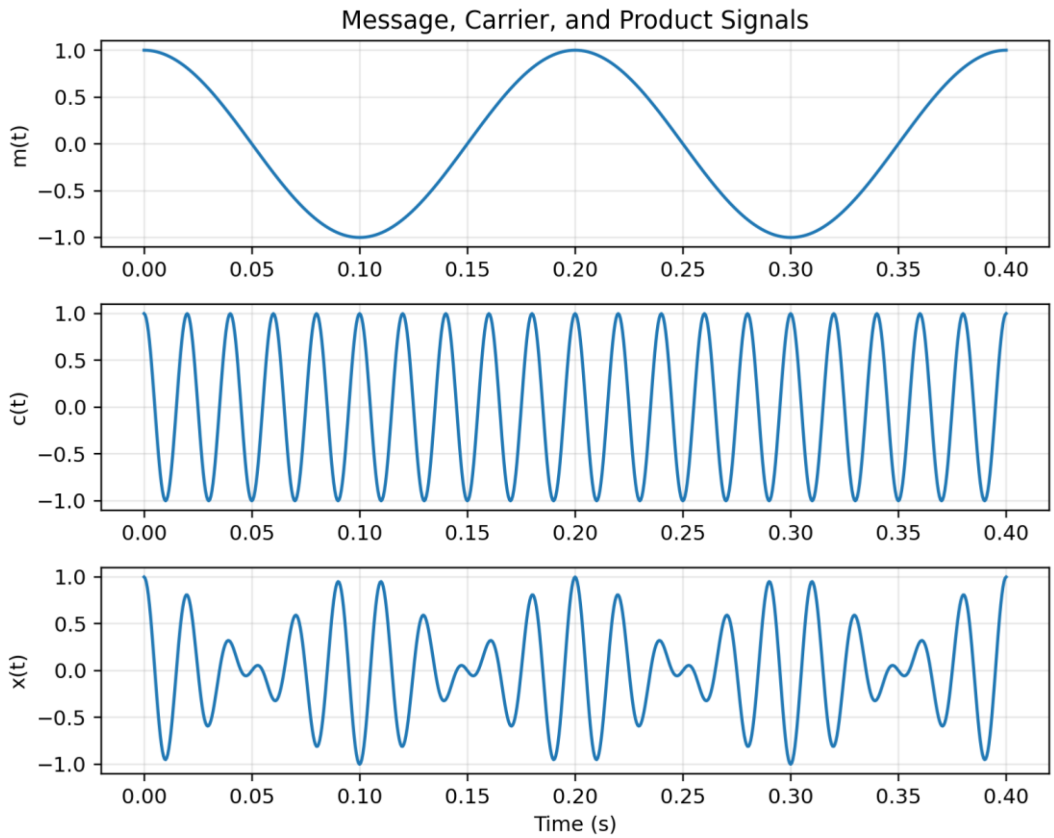

Fig. 3.1. Message, carrier, and product signals in the time domain.

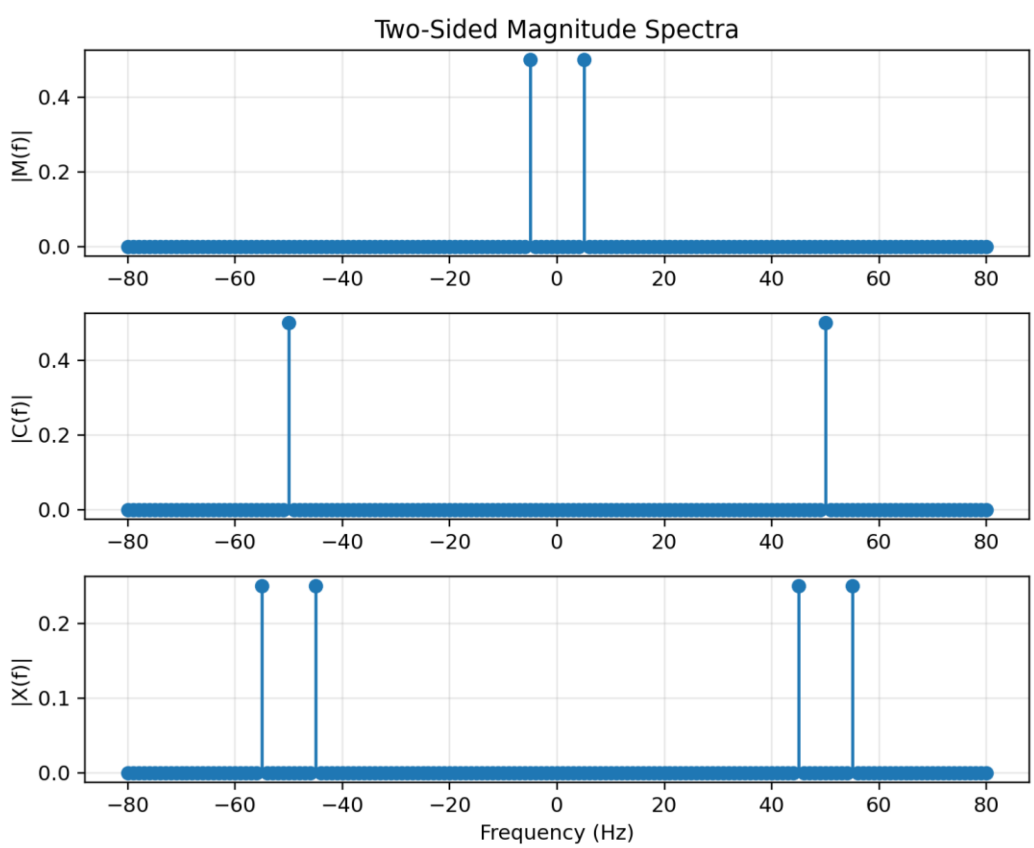

Fig. 3.2. Frequency translation produced by signal multiplication.

2.3. Conventional Amplitude Modulation

In conventional amplitude modulation, the amplitude of a sinusoidal carrier is varied according to the instantaneous value of the message signal. For a normalized single-tone message, the AM signal is expressed as

$$s\_AM(t) = A\_c[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f\_ct)$$

where μ is the modulation index. Expanding the signal gives

$$s\_AM(t) = A\_c \cos(2\pi f\_ct) + (\mu A\_c/2)\cos[2\pi(f\_c - f_m)t] + (\mu A\_c/2)\cos[2\pi(f\_c + f_m)t]$$

The spectrum consequently contains:

- a carrier component at $f = \pm f\_c$,
- a lower sideband at $f = \pm(f\_c - f_m)$,
- an upper sideband at $f = \pm(f\_c + f_m)$.

$$BW\_AM = (f\_c + f_m) - (f\_c - f_m) = 2f_m$$

For $0 \le \mu \le 1$, the envelope follows the message without crossing zero. At $\mu = 1$, the signal is fully modulated. For $\mu > 1$, overmodulation occurs and envelope distortion is expected.

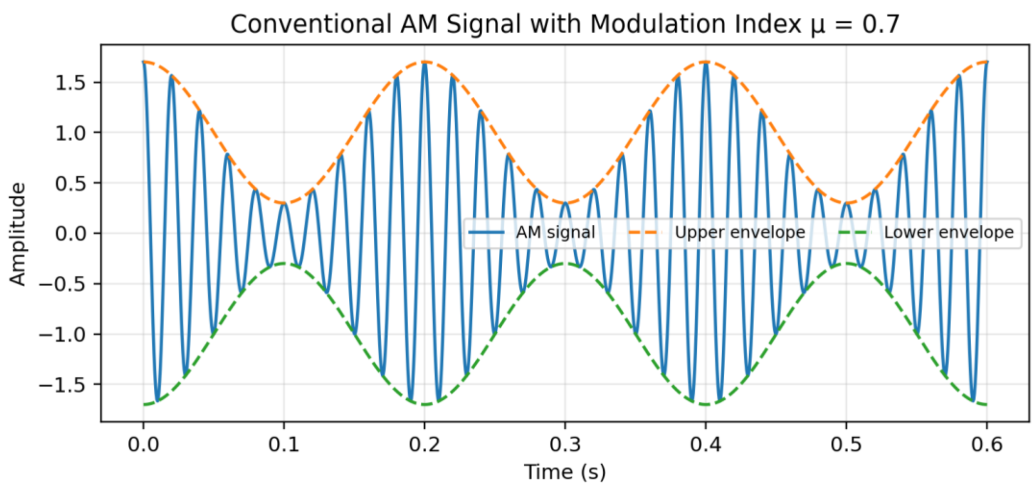

Fig. 3.3. Conventional AM waveform and its envelope for μ = 0.7.

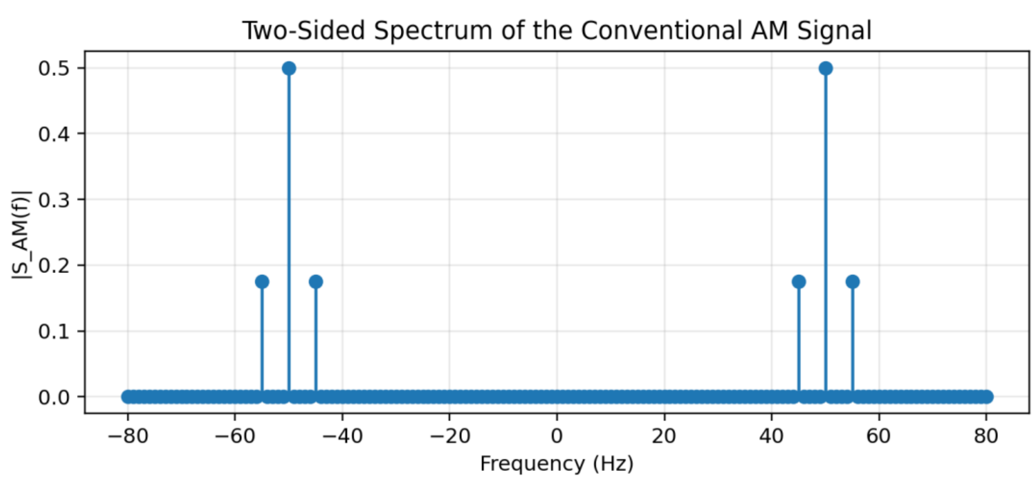

Fig. 3.4. Carrier and sideband components of the conventional AM spectrum.

2.4. Modulation Index and AM Power

For a single-tone AM signal, the modulation index may be determined from the maximum and minimum envelope values:

$$\mu = (A\_max - A\_min)/(A\_max + A\_min)$$

For a resistive load R, the carrier power, total sideband power, and total transmitted power are

$$P\_c = A\_c^2/(2R)$$

$$P\_SB = (\mu^2/2)P\_c$$

$$P\_T = P\_c(1 + \mu^2/2)$$

The fraction of the total power carried by the sidebands is

$$\eta = P\_SB/P\_T = \mu^2/(2 + \mu^2)$$

2.5. Double-Sideband Suppressed-Carrier Modulation

A DSB-SC signal is obtained by directly multiplying the message and carrier signals:

$$s\_DSB\text{-}SC(t) = A\_c\, m(t)\cos(2\pi f\_ct)$$

For $m(t) = \cos(2\pi f_m t)$,

$$s\_DSB\text{-}SC(t) = (A\_c/2)\cos[2\pi(f\_c - f_m)t] + (A\_c/2)\cos[2\pi(f\_c + f_m)t]$$

The spectrum contains only the upper and lower sidebands; there is no discrete component at f_c because an unmodulated carrier term is not added. The DSB-SC bandwidth is the same as the conventional AM bandwidth for the same message signal:

$$BW\_DSB\text{-}SC = 2f_m$$

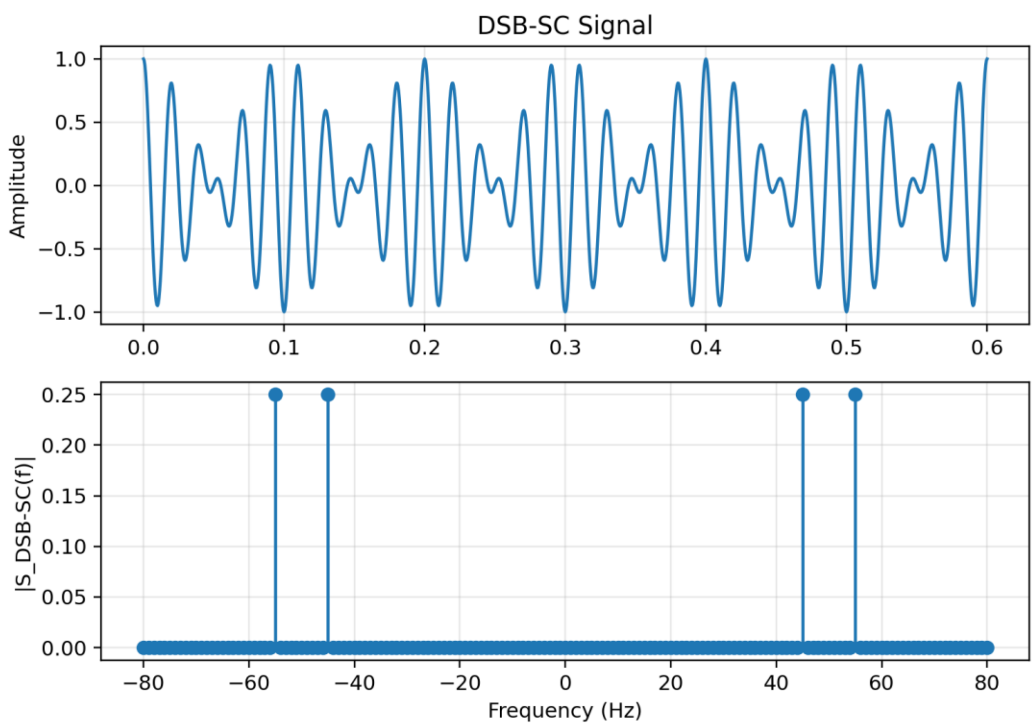

Fig. 3.5. Time-domain waveform and spectrum of a DSB-SC signal.

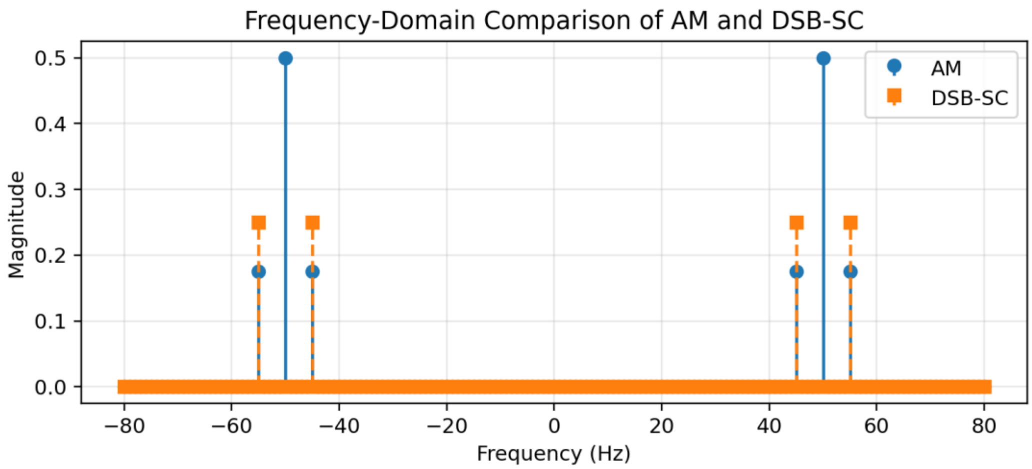

Fig. 3.6. Spectral comparison of conventional AM and DSB-SC modulation.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Script Editor
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, abs and max/min operations
- fft, fftshift and length
- plot, stem and subplot
- grid, xlabel, ylabel, title, legend and xlim

4. Preliminary (PreLab)

Complete the following calculations and sketches before attending the laboratory.

4.1. Frequency Translation

$$m(t) = \cos(2\pi 5t), \quad c(t) = \cos(2\pi 50t)$$

$$x(t) = m(t)c(t)$$

1. Use the product-to-sum identity to express x(t) as the sum of two cosine signals.
2. Determine all positive and negative frequency components expected in X(f).
3. Determine the theoretical magnitude of each line in a normalized two-sided FFT spectrum.
4. Sketch M(f), C(f), and X(f) on separate frequency axes.

4.2. Conventional AM Signal

$$s\_AM(t) = [1 + 0.7\cos(2\pi 5t)]\cos(2\pi 50t)$$

1. Determine the carrier, lower-sideband, and upper-sideband frequencies.
2. Expand the AM signal into three cosine terms and determine their amplitudes.
3. Calculate the transmission bandwidth.

4. Determine the maximum and minimum envelope amplitudes.
5. Verify the modulation index using $\mu = (A_{max} - A_{min})/(A_{max} + A_{min})$.
6. Calculate the theoretical AM efficiency $\eta = \mu^2/(2 + \mu^2)$.

4.3. DSB-SC Signal

$$s_{DSB\text{-}SC}(t) = \cos(2\pi 5t)\cos(2\pi 50t)$$

1. Expand the signal into its upper- and lower-sideband components.
2. Determine the expected spectral frequencies and amplitudes.
3. Calculate the DSB-SC bandwidth.
4. Explain theoretically why no component is expected at 50 Hz.
5. State one frequency-domain difference between conventional AM and DSB-SC.

Table 3.1. Preliminary frequency and amplitude calculations.

Signal	Component	Frequency (Hz)	Theoretical two-sided magnitude
m(t)	Positive message component		
m(t)	Negative message component		
c(t)	Positive carrier component		
c(t)	Negative carrier component		
m(t)c(t)	Lower sideband		
m(t)c(t)	Upper sideband		
AM	Carrier		
AM	Lower sideband		
AM	Upper sideband		
DSB-SC	Lower sideband		
DSB-SC	Upper sideband		

5. Procedure

Part 1: Analysis of Signal Multiplication and Frequency Translation

1. Open MATLAB and create a new script file.
2. Set the sampling frequency to $F_s = 2000$ Hz and define the observation interval as $0 \le t < 1$ s.
3. Generate the message signal $m(t) = \cos(2\pi 5t)$.
4. Generate the carrier signal $c(t) = \cos(2\pi 50t)$.
5. Generate the product signal $x(t) = m(t)c(t)$.
6. Plot the time-domain waveforms of m(t), c(t), and x(t) using subplot(3,1,...).
7. Calculate the normalized, centered FFT of each signal.
8. Construct the corresponding two-sided frequency axis.
9. Plot $|M(f)|$, $|C(f)|$, and $|X(f)|$ using subplot(3,1,...).
10. Identify the frequencies and magnitudes of the dominant spectral components.
11. Compare the measured product frequencies with $f_c - f_m$ and $f_c + f_m$.

```
Fs = 2000;
t = 0:1/Fs:1-1/Fs;
fm = 5;  fc = 50;
m = cos(2*pi*fm*t);
c = cos(2*pi*fc*t);
x = m .* c;
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);

M = fftshift(fft(m))/N;
C = fftshift(fft(c))/N;
X = fftshift(fft(x))/N;
```

Part 2: Analysis of a Conventional AM Signal

1. Use f_m = 5 Hz, f_c = 50 Hz, and μ = 0.7.
2. Generate s_AM(t) = [1 + μcos(2πf_mt)]cos(2πf_ct).
3. Plot the AM signal over a time interval long enough to observe several message periods.
4. Plot the theoretical upper and lower envelopes, ±[1 + μcos(2πf_mt)], together with the AM signal.
5. Determine the maximum and minimum envelope amplitudes directly from the generated data.
6. Calculate the measured modulation index from the envelope values.
7. Calculate and plot the normalized two-sided FFT of the AM signal.
8. Identify the carrier and sideband frequencies and measure their spectral magnitudes.
9. Calculate the measured transmission bandwidth.
10. Repeat the time-domain analysis for μ = 0.3, μ = 1.0, and μ = 1.3.
11. Compare the envelope behavior for under-modulation, full modulation, and overmodulation.

```
mu = 0.7;
s_am = (1 + mu*cos(2*pi*fm*t)) .* cos(2*pi*fc*t);
S_AM = fftshift(fft(s_am))/N;
upper_env = 1 + mu*cos(2*pi*fm*t);
lower_env = -upper_env;
```

Part 3: Analysis of a DSB-SC Modulated Signal

1. Generate s_DSB-SC(t) = cos(2πf_mt)cos(2πf_ct) using f_m = 5 Hz and f_c = 50 Hz.
2. Plot the DSB-SC signal in the time domain.
3. Calculate and plot its normalized two-sided FFT.
4. Identify the lower- and upper-sideband components.
5. Verify that no discrete carrier component appears at ±50 Hz.
6. Calculate the measured DSB-SC bandwidth.
7. Plot the AM and DSB-SC spectra on the same frequency axis.
8. Compare the carrier and sideband magnitudes of the two modulation methods.
9. Explain why the sign reversal of the DSB-SC waveform is not equivalent to a conventional AM envelope.

```
s_dsb = cos(2*pi*fm*t) .* cos(2*pi*fc*t);
S_DSB = fftshift(fft(s_dsb))/N;
```

6. Evaluation of Results

Tabulate the theoretical and MATLAB-obtained results. Percentage error is calculated as

$$\%\ \text{Error} = |(\text{MATLAB value} - \text{Theoretical value})/(\text{Theoretical value})| \times 100$$

Table 3.2. Signal multiplication and frequency-translation results.

Quantity	Theoretical value	MATLAB value	Percentage error
Message frequency			
Carrier frequency			
Lower product frequency			
Upper product frequency			
Magnitude of each product component			

Table 3.3. Conventional AM results for μ = 0.7.

Quantity	Theoretical value	MATLAB value	Percentage error
Carrier frequency			
Lower-sideband frequency			
Upper-sideband frequency			
Carrier spectral magnitude			
Each sideband magnitude			

Quantity	Theoretical value	MATLAB value	Percentage error
Maximum envelope amplitude			
Minimum envelope amplitude			
Modulation index			
Bandwidth			
AM efficiency			

Table 3.4. Effect of the modulation index on the AM waveform.

Modulation index	A_max	A_min	Envelope condition	Observation
0.3				
0.7				
1.0				
1.3				

Table 3.5. DSB-SC results and comparison with conventional AM.

Quantity	DSB-SC result	Conventional AM result
Carrier component at f_c		
Lower-sideband frequency		
Upper-sideband frequency		
Magnitude of each sideband		
Transmission bandwidth		
Presence of a directly observable envelope		

Questions

1. Why does multiplying two cosine signals produce sum and difference frequencies?
2. How does the product spectrum demonstrate frequency translation?
3. Which spectral component of a conventional AM signal carries no message information?
4. Why are the upper- and lower-sideband amplitudes proportional to the modulation index?
5. What changes occur in the AM spectrum when μ is increased while f_m and f_c remain constant?
6. What happens to the AM envelope when $\mu > 1$?
7. Why does the DSB-SC spectrum not contain a line at f_c?
8. Why do conventional AM and DSB-SC have the same bandwidth for the same message signal?
9. Compare the transmitted-power utilization of conventional AM and DSB-SC.
10. Explain the small differences that may occur between theoretical line magnitudes and FFT results.
11. What conditions on the sampling frequency and observation duration are required to identify the spectral components accurately?

Experiment 4: Square-Law Modulation and Detection

1. Objective of the Experiment

The objective of this experiment is to investigate the operation of square-law devices in amplitude-modulation and demodulation applications using MATLAB. A nonlinear element is first modeled by a second-order input-output relation. Students will examine how the squaring operation generates harmonic, sum-frequency, and difference-frequency components and how a band-pass filter can select the components required for amplitude modulation. In the second part, an amplitude-modulated signal is applied to a square-law detector, and a low-pass filter is used to recover its baseband information. The theoretical frequency components are compared with normalized FFT results in order to relate nonlinear signal processing to practical communication-system blocks.

At the end of the experiment, students are expected to:

- Generate multitone message, carrier, nonlinear input, and square-law output signals in MATLAB.
- Expand a square-law model and classify the resulting spectral products.
- Identify the original, harmonic, sum, and difference frequencies in an FFT spectrum.
- Explain how a band-pass filter selects carrier and sideband components around the carrier frequency.
- Apply square-law detection to a conventional AM signal.
- Design a low-pass filter that retains the desired message frequencies while rejecting carrier-related components.
- Compare the original and recovered message signals in the time and frequency domains.
- Assess nonlinear distortion and demodulation performance using quantitative error measures.

2. Theoretical Background

2.1. Nonlinear and Square-Law Devices

An ideal linear device produces an output that is directly proportional to its input. Practical electronic devices, however, may exhibit nonlinear behavior. When the input amplitude is restricted to a suitable operating range, the nonlinear characteristic can be approximated by the first terms of a power-series expansion. A second-order or square-law approximation is written as

$$y(t) = a_1x(t) + a_2x^2(t)$$

where a_1 represents the linear coefficient and a_2 represents the second-order nonlinear coefficient. The $x^2(t)$ term creates new frequency components that are not present in the original input. These components can be used intentionally for modulation and detection, but they may also produce unwanted nonlinear distortion.

2.2. Square-Law Modulation

For square-law modulation, the message and carrier signals are added before the nonlinear element:

$$x(t) = m(t) + c(t)$$

Substitution into the square-law relation gives

$$y(t) = a_1m(t) + a_1c(t) + a_2m^2(t) + 2a_2m(t)c(t) + a_2c^2(t)$$

The term $2a_2m(t)c(t)$ contains the product of the message and carrier and therefore translates the message spectrum to frequencies around the carrier. The term $a_1c(t)$ provides a carrier component. If a band-pass filter centered at the carrier frequency rejects the baseband, low-frequency mixing products, and carrier harmonics, the selected output has the form

$$s\_BPF(t) = a_1A\_c\cos(2\pi f\_ct) + 2a_2A\_c\, m(t)\cos(2\pi f\_ct)$$

$$s\_BPF(t) = A\_c[a_1 + 2a_2m(t)]\cos(2\pi f\_ct)$$

This expression has the structure of a conventional amplitude-modulated signal. The carrier and translated message components can therefore be selected from the nonlinear output by using a suitable band-pass filter.

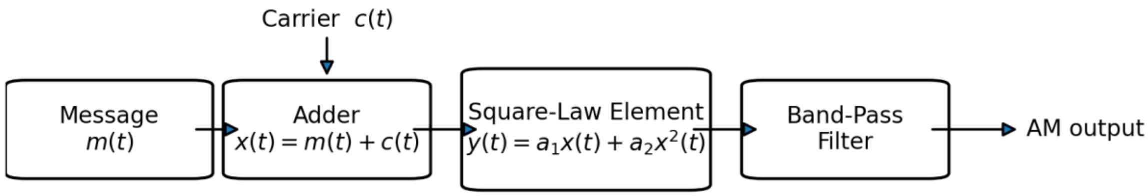

Fig. 4.1. Functional representation of a square-law modulator followed by a band-pass filter.

2.3. Spectral Products Generated by Squaring

The frequency products generated by the square-law term can be determined using the trigonometric identities

$$\cos^2(2\pi ft) = ½ + ½\cos(2\pi{\cdot}2ft)$$

$$2\cos(2\pi f_1t)\cos(2\pi f_2t) = \cos[2\pi(f_1 - f_2)t] + \cos[2\pi(f_1 + f_2)t]$$

Consequently, squaring a multitone signal produces:

- a DC component from the squared sinusoidal terms,
- second harmonics at twice each original frequency,
- difference-frequency components at $|f_i - f_j|$,
- sum-frequency components at $f_i + f_j$.

For the message frequencies 17 Hz and 29 Hz and the carrier frequency 165 Hz used in this experiment, the square-law output contains components at 12, 34, 46, 58, 136, 148, 182, 194, and 330 Hz, in addition to the original linear components at 17, 29, and 165 Hz and a DC term. The components located near 165 Hz are the carrier and its modulation sidebands.

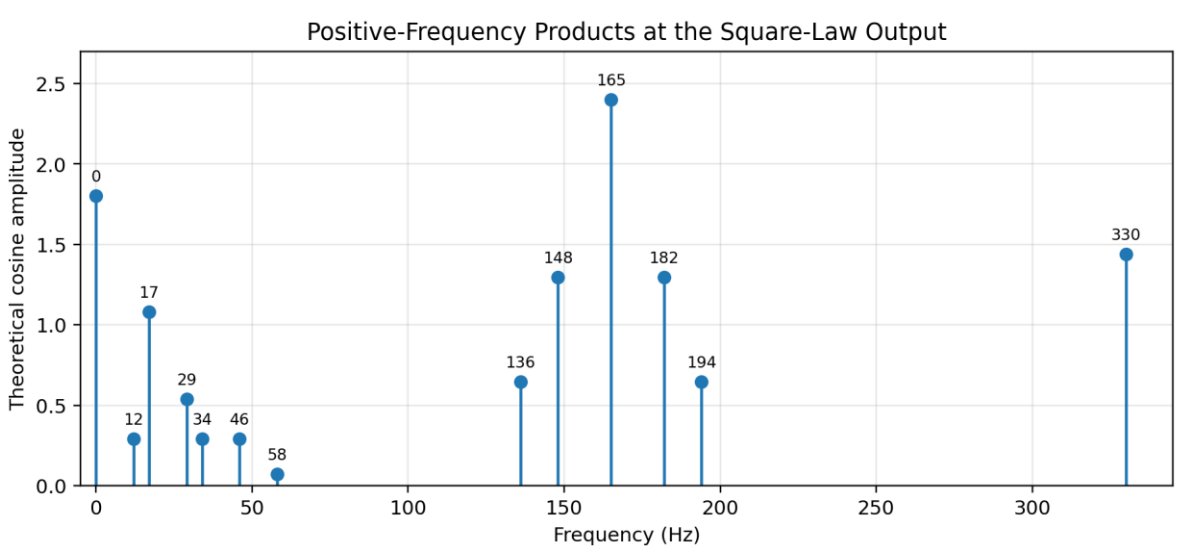

Fig. 4.2. Theoretical positive-frequency components for the square-law modulator parameters used in the experiment.

2.4. Square-Law Detection

A square-law detector uses the same nonlinear principle to recover the baseband information from an AM signal. Consider an AM signal expressed as

$$s\_AM(t) = A(t)cos(2\pi f\_ct)$$

where A(t) is the slowly varying envelope. Squaring the AM signal gives

$$y(t) = s^2\_AM(t) = A^2(t)cos^2(2\pi f\_ct)$$

$$y(t) = ½A^2(t) + ½A^2(t)cos(4\pi f\_ct)$$

The first term is a low-frequency component that contains the message information, whereas the second term is centered around twice the carrier frequency. A low-pass filter suppresses the high-frequency term and produces

$$y\_LP(t) = ½A^2(t)$$

When the envelope variation is sufficiently small, the terms that are linear in the message dominate the recovered waveform. For larger modulation depths or multitone messages, $A^2(t)$ also contains harmonic and intermodulation products. Therefore, the low-pass cutoff frequency must be chosen carefully, and the recovered waveform may contain nonlinear distortion even after carrier-related terms have been rejected.

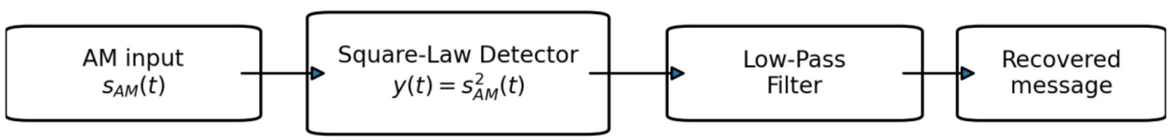

Fig. 4.3. Square-law detector and low-pass filtering stages.

2.5. Frequency Selection and FFT Analysis

A band-pass filter is used in the modulator to retain components around the carrier frequency, while a low-pass filter is used in the detector to retain the desired message frequencies. In this experiment, the modulator band-pass range is selected to include the carrier and all four sidebands generated by the two message tones. For detection, the desired message frequencies are 24 Hz and 38 Hz. The nearest undesired second-order baseband component occurs at 48 Hz; therefore, a cutoff frequency between 38 Hz and 48 Hz provides a useful separation region.

For a sampled signal of length N and sampling frequency F_s, the normalized centered FFT is obtained from

$$X\_c[k] = fftshift\{fft(x[n])\}/N$$

The frequency spacing is $\Delta f = F\_s/N$. A sufficiently long observation interval should be used so that the spectral lines coincide with FFT bins and the predicted products can be identified accurately.

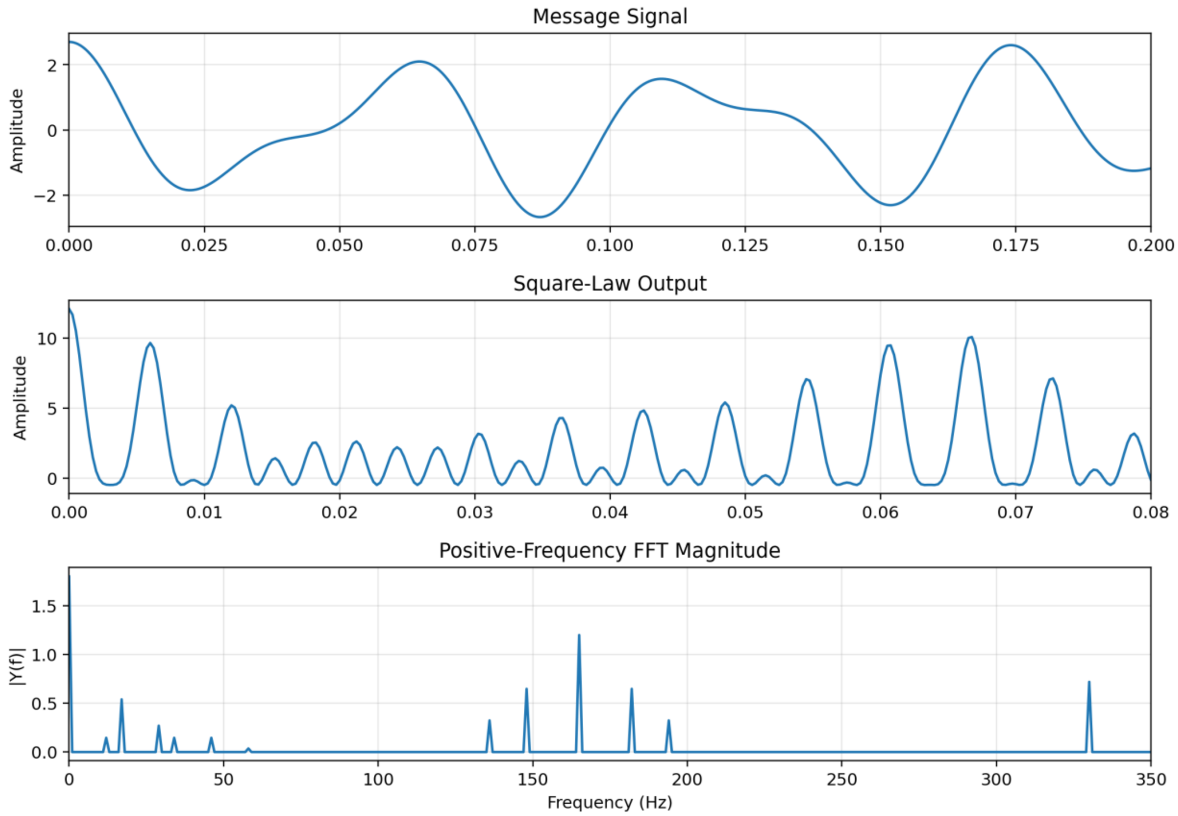

Fig. 4.4. Example time-domain and FFT results for the square-law modulator.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Script Editor
- MATLAB Signal Processing Toolbox or equivalent frequency-domain filtering method
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, mean, max, abs and element-wise power operations
- fft, ifft, fftshift, ifftshift, fftfreq-equivalent frequency-axis construction and length
- plot, stem, subplot, grid, xlabel, ylabel, title, legend and xlim
- butter, filtfilt or an equivalent FFT-domain filter mask

4. Preliminary (PreLab)

Complete the following calculations and classifications before attending the laboratory.

4.1. Square-Law Modulator

$$m(t) = 1.8\cos(2\pi 17t) + 0.9\cos(2\pi 29t)$$

$$c(t) = 4\cos(2\pi 165t)$$

$$x(t) = m(t) + c(t), \quad y(t) = 0.6x(t) + 0.18x^2(t)$$

1. Expand $x^2(t)$ completely using trigonometric identities.
2. Determine the DC component and all positive-frequency components expected in y(t).
3. Classify each component as an original linear term, second harmonic, sum frequency, or difference frequency.
4. Calculate the theoretical cosine amplitude associated with every component.
5. Identify the carrier and sideband frequencies that should be passed by a band-pass filter centered at 165 Hz.
6. Propose lower and upper cutoff frequencies that retain 136, 148, 165, 182, and 194 Hz while rejecting the adjacent unwanted products.

Table 4.1. Preliminary analysis of the square-law modulator output.

Frequency (Hz)	Origin / expression	Classification	Theoretical amplitude
0		DC	
12	|29 − 17|	Difference	
17		Linear	
29		Linear	
34	2 × 17	Second harmonic	
46	17 + 29	Sum	
58	2 × 29	Second harmonic	
136	165 − 29	Lower sideband	
148	165 − 17	Lower sideband	
165		Carrier / linear	
182	165 + 17	Upper sideband	
194	165 + 29	Upper sideband	
330	2 × 165	Carrier harmonic	

4.2. Square-Law Detector

$$s\_AM(t) = [1 + 0.72\cos(2\pi 24t) + 0.35\cos(2\pi 38t)]\cos(2\pi 240t)$$

$$y(t) = s^2\_AM(t)$$

1. Express y(t) as the sum of a low-frequency term and a component centered around 2f_c.
2. Expand the low-frequency term $\frac{1}{2}A^2(t)$, where $A(t) = 1 + 0.72\cos(2\pi 24t) + 0.35\cos(2\pi 38t)$.
3. Determine the expected baseband components at DC, message, harmonic, sum, and difference frequencies.
4. Identify the desired message frequencies and the nearest undesired baseband frequency.
5. Select a low-pass cutoff frequency that passes 24 Hz and 38 Hz while attenuating the component at 48 Hz.

6. Predict whether the filtered waveform will be an exact scaled copy of the original message and justify the answer.

Table 4.2. Preliminary baseband products at the square-law detector output.

Frequency (Hz)	Classification	Desired / undesired	Theoretical amplitude
0	DC	Remove after filtering	
14	Difference: 38 − 24	Undesired distortion	
24	Message component	Desired	
38	Message component	Desired	
48	Second harmonic: 2 × 24	Undesired distortion	
62	Sum: 24 + 38	Undesired distortion	
76	Second harmonic: 2 × 38	Undesired distortion	

5. Procedure

Part 1: Square-Law Modulator Analysis

1. Open MATLAB and create a new script file.
2. Set the sampling frequency to F_s = 4000 Hz and define the observation interval as $0 \le t < 1$ s.
3. Generate the message signal $m(t) = 1.8\cos(2\pi 17t) + 0.9\cos(2\pi 29t)$.
4. Generate the carrier signal $c(t) = 4\cos(2\pi 165t)$.
5. Form the nonlinear-element input $x(t) = m(t) + c(t)$.
6. Calculate the square-law output $y(t) = 0.6x(t) + 0.18x^2(t)$ using element-wise operations.
7. Plot m(t), c(t), x(t), and y(t) in separate subplots. Select time-axis limits that clearly show the message and carrier behavior.
8. Calculate the normalized centered FFT of m(t), c(t), x(t), and y(t).
9. Plot the positive-frequency portion of |Y(f)| over 0-350 Hz.
10. Identify the frequencies and magnitudes of all significant components and record them in Table 4.3.
11. Compare the FFT peaks with the theoretical harmonic, sum, and difference frequencies calculated in the PreLab.
12. Design a band-pass filter that passes approximately 130-200 Hz.
13. Apply the band-pass filter to y(t) and denote the result by s_BPF(t).
14. Plot the filtered waveform and spectrum.
15. Verify that the filtered output contains the carrier at 165 Hz and sidebands at 136, 148, 182, and 194 Hz.
16. Comment on which terms of the square-law expansion are removed and which are retained by the filter.

```
Fs = 4000;
t = 0:1/Fs:1-1/Fs;
m = 1.8*cos(2*pi*17*t) + 0.9*cos(2*pi*29*t);
c = 4*cos(2*pi*165*t);
x = m + c;
y = 0.6*x + 0.18*(x.^2);
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
Y = fftshift(fft(y))/N;

[bBP,aBP] = butter(4,[130 200]/(Fs/2),'bandpass');
s_bpf = filtfilt(bBP,aBP,y);
S_BPF = fftshift(fft(s_bpf))/N;
```

Part 2: Square-Law Detection and Low-Pass Filtering

1. Set the sampling frequency to F_s = 6000 Hz and define the observation interval as $0 \leq t < 1$ s.
2. Generate the baseband message $m(t) = 0.72\cos(2\pi 24t) + 0.35\cos(2\pi 38t)$.
3. Generate the AM signal $s_{AM}(t) = [1 + m(t)]\cos(2\pi 240t)$.
4. Plot the AM signal and its upper and lower envelopes.
5. Calculate the square-law detector output $y_D(t) = s^2_{AM}(t)$.
6. Calculate and plot the normalized centered FFTs of $s_{AM}(t)$ and $y_D(t)$.
7. Identify the low-frequency products and the components centered around $2f_c$ = 480 Hz.
8. Design a low-pass filter with a cutoff frequency of approximately 42 Hz.
9. Apply the low-pass filter to $y_D(t)$.
10. Remove the DC component from the filtered signal by subtracting its mean value.
11. Scale the recovered waveform when necessary and compare it with the original message over the same time interval.
12. Plot the spectra before and after low-pass filtering.
13. Record the magnitudes of the 14, 24, 38, 48, 62, and 76 Hz components before and after filtering.
14. Calculate the mean-squared error or normalized root-mean-square error between the original and recovered message waveforms.
15. Comment on the remaining nonlinear distortion and the effect of the cutoff frequency on demodulation performance.

```
Fs = 6000;
t = 0:1/Fs:1-1/Fs;
m = 0.72*cos(2*pi*24*t) + 0.35*cos(2*pi*38*t);
s_am = (1 + m).*cos(2*pi*240*t);
y_d = s_am.^2;
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
Y_D = fftshift(fft(y_d))/N;

fc_lp = 42;
[bLP,aLP] = butter(6,fc_lp/(Fs/2),'low');
y_lp = filtfilt(bLP,aLP,y_d);
m_rec = y_lp - mean(y_lp);
```

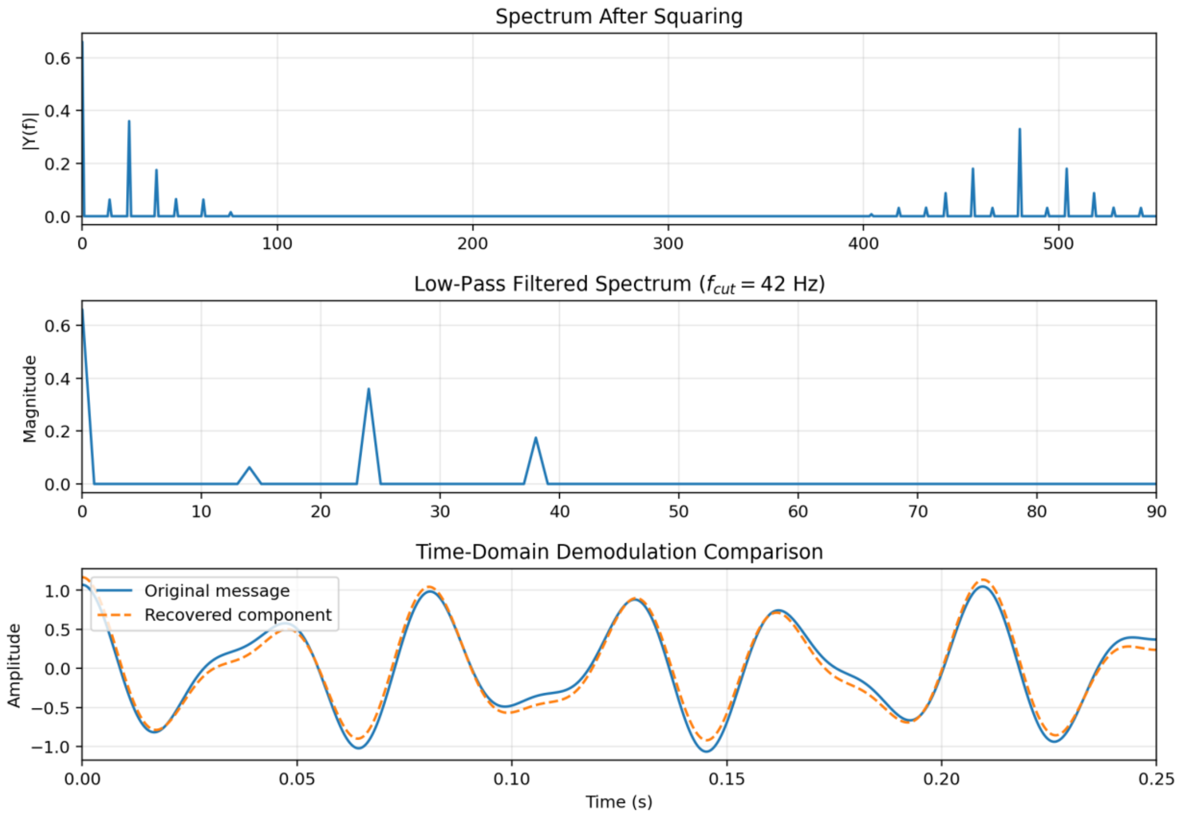

Fig. 4.5. Example spectrum and time-domain results for square-law detection and low-pass filtering.

6. Evaluation of Results

Tabulate the theoretical and MATLAB-obtained results. Percentage error is calculated as

$$\%\ Error = |(MATLAB\ value - Theoretical\ value)/(Theoretical\ value)| \times 100$$

Table 4.3. Measured square-law modulator output components.

Frequency (Hz)	Classification	Theoretical amplitude	FFT amplitude	Error (%)
0	DC			
12	Difference			
17	Linear			
29	Linear			
34	Second harmonic			
46	Sum			
58	Second harmonic			
136	Lower sideband			
148	Lower sideband			
165	Carrier			

Frequency (Hz)	Classification	Theoretical amplitude	FFT amplitude	Error (%)
182	Upper sideband			
194	Upper sideband			
330	Carrier harmonic			

Table 4.4. Components selected by the band-pass filter.

Component	Frequency (Hz)	Before BPF magnitude	After BPF magnitude	Retained / rejected
Lower sideband 2	136			
Lower sideband 1	148			
Carrier	165			
Upper sideband 1	182			
Upper sideband 2	194			
Carrier harmonic	330			

Table 4.5. Square-law detector baseband results.

Frequency (Hz)	Expected origin	Before LPF magnitude	After LPF magnitude	Desired / distortion
0	DC			
14	Difference			
24	Message			
38	Message			
48	Second harmonic			
62	Sum			
76	Second harmonic			

Table 4.6. Demodulation-performance summary.

Parameter	Value
Sampling frequency, F_s	
Low-pass cutoff frequency	
Recovered 24 Hz magnitude	
Recovered 38 Hz magnitude	
Residual 14 Hz magnitude	

Parameter	Value
Mean value removed from filter output	
MSE or NRMSE	
Correlation coefficient	

Answer the following questions:

1. Why does the square-law operation generate frequencies that are absent from the input signal?
2. Which output components are classified as harmonics, and how are their frequencies determined?
3. Which frequency products result from interactions between the two message tones?
4. Which products result from interactions between each message tone and the carrier?
5. Why can a band-pass filter centered around 165 Hz produce an amplitude-modulated output?
6. How would changing the band-pass limits affect the transmitted carrier and sidebands?
7. Why does the square-law detector output contain a group of components around 480 Hz?
8. Why are 24 Hz and 38 Hz considered the desired detector outputs, while 14 Hz, 48 Hz, 62 Hz, and 76 Hz represent nonlinear distortion?
9. Why is a cutoff frequency between 38 Hz and 48 Hz appropriate for this detector example?
10. Is the recovered waveform an exact scaled copy of the original message? Explain any difference using the expansion of $A^2(t)$.
11. How would a larger message amplitude affect the amount of square-law distortion?
12. Compare the theoretical and MATLAB-obtained spectra and discuss the effects of observation time, FFT resolution, filter transition band, and numerical implementation.

Experiment 5: Envelope Detection of Conventional AM Signals

1. Objective of the Experiment

The objective of this experiment is to investigate the generation and envelope detection of conventional amplitude-modulated (AM) signals using MATLAB. Students will first relate the message, carrier, sidebands, and mathematical envelope of an AM waveform. The signal will then be demodulated using both an ideal analytic-signal envelope and a practical diode-capacitor-resistor envelope-detector model.

The experiment emphasizes the selection of the detector time constant, the influence of carrier ripple and diagonal clipping, and the condition that the modulation index must not exceed unity for undistorted envelope detection. Time-domain and frequency-domain results will be compared quantitatively.

At the end of the experiment, students are expected to:

- Generate a conventional AM signal and identify its carrier and sideband components.
- Calculate the modulation index from the maximum and minimum envelope values.
- Obtain the ideal envelope using the analytic signal in MATLAB.
- Implement a discrete-time diode-RC peak detector.
- Explain why the detector time constant must be large relative to the carrier period but small relative to the message period.
- Distinguish carrier ripple from diagonal clipping.
- Demonstrate the failure of envelope detection under overmodulation.
- Compare the original and recovered message signals using spectral and error measures.

2. Theoretical Background

2.1. Conventional Amplitude Modulation

In conventional amplitude modulation, the amplitude of a high-frequency carrier is varied according to a lower-frequency message signal. For a normalized message m_n(t), satisfying |m_n(t)| ≤ 1, the AM signal is

$$s\_AM(t) = Ac[1 + \mu m\_n(t)]cos(2\pi fct)$$

where Ac is the carrier amplitude, fc is the carrier frequency, and μ is the modulation index. The slowly varying envelope is

$$E(t) = Ac[1 + \mu m\_n(t)]$$

For a single-tone message m_n(t)=cos(2πfmt), the AM waveform may be expanded as

$$s\_AM(t) = Ac\ cos(2\pi fct) + (\mu Ac/2)cos[2\pi(fc\text{-}fm)t] + (\mu Ac/2)cos[2\pi(fc\text{+}fm)t]$$

Thus, the spectrum contains a carrier at fc and two sidebands at fc-fm and fc+fm. The transmission bandwidth is

$$BW\_AM = 2fm$$

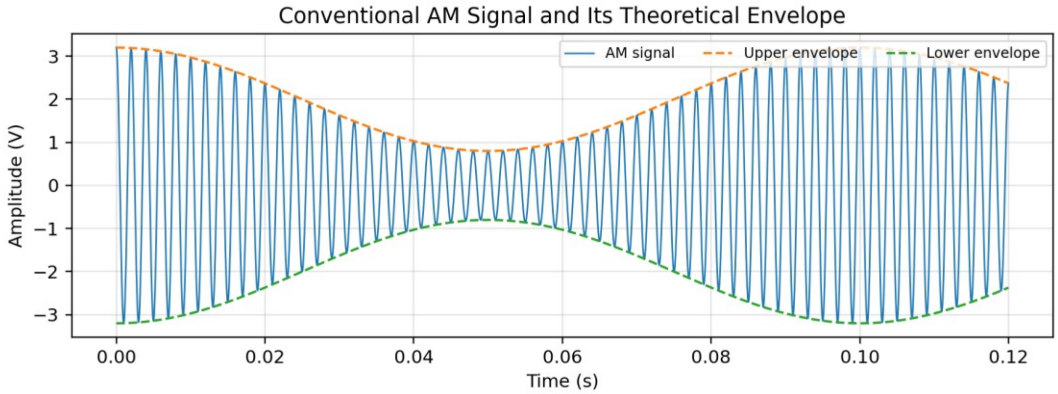

Fig. 5.1. Conventional AM waveform and its theoretical upper and lower envelopes.

2.2. Modulation Index and Overmodulation

When the envelope can be measured directly, the modulation index is obtained from its maximum and minimum values:

$$\mu = (Emax - Emin)/(Emax + Emin)$$

For $0 \leq \mu < 1$, the envelope remains positive and follows the message waveform. At μ=1, the envelope reaches zero at its minimum. For μ>1, the envelope changes sign and the AM signal is overmodulated. A simple envelope detector follows the magnitude of the RF waveform rather than the signed mathematical envelope; therefore, it cannot recover the message without distortion when overmodulation occurs.

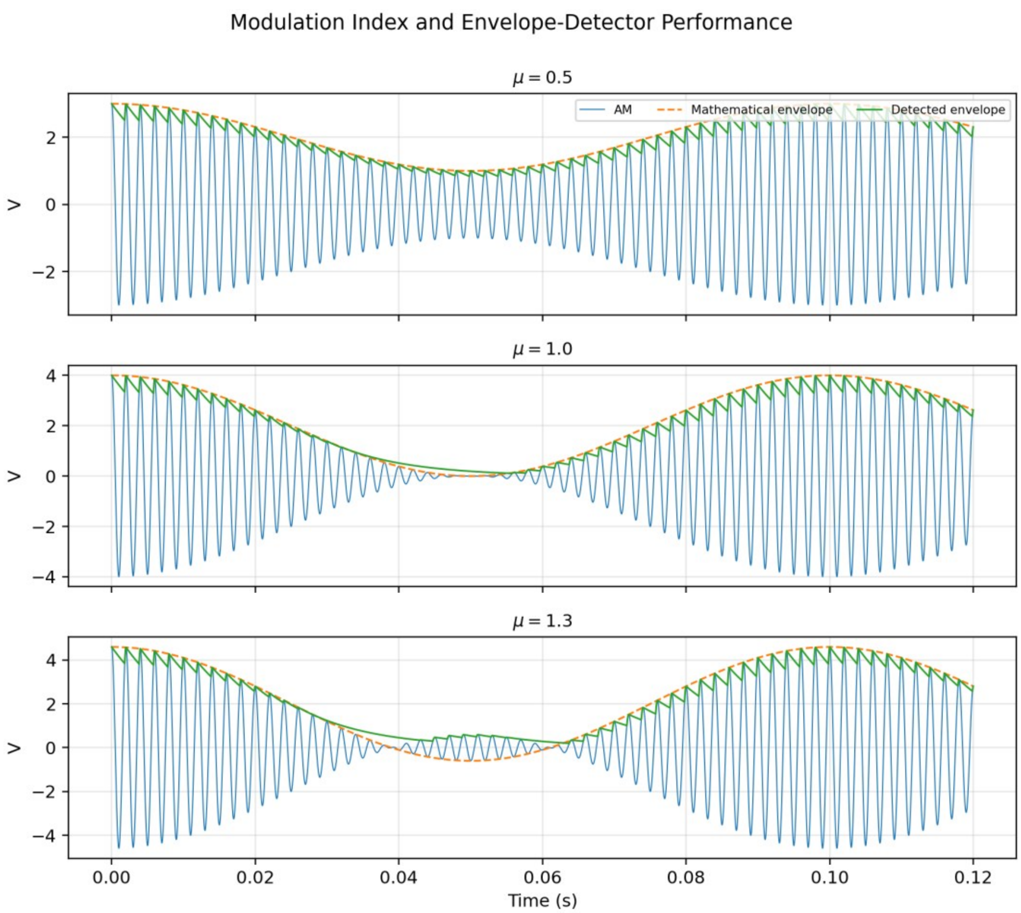

Fig. 5.2. Effect of the modulation index on the AM waveform and detected envelope.

2.3. Ideal Envelope Detection

The ideal envelope of a real bandpass signal can be obtained from its analytic signal. If H{s(t)} denotes the Hilbert transform, the analytic signal and its envelope are

$$z(t) = s(t) + jH\{s(t)\}$$

$$E\_ideal(t) = |z(t)|$$

MATLAB implements this operation using abs(hilbert(s)). Ideal envelope extraction is useful as a reference because it does not include diode threshold, charging delay, carrier ripple, or RC-discharge distortion.

2.4. Diode-RC Envelope Detector

A practical envelope detector consists of a rectifying diode followed by a capacitor and load resistor. When the instantaneous rectified input exceeds the capacitor voltage, the diode conducts and the capacitor charges rapidly toward the new peak. When the input decreases, the diode becomes reverse biased and the capacitor discharges through the load resistance with the time constant

$$\tau = R\_LC$$

For correct tracking, the charging time constant associated with the source and diode must be short, while the discharge time constant must satisfy the approximate condition

$$Tc = 1/fc \ll \tau \ll Tm = 1/fm$$

Equivalently, for a message bandwidth W, the lecture-note condition can be expressed as 1/fc ≪ R_LC ≪ 1/W. A time constant that is too short allows excessive carrier ripple. A time constant that is too long prevents the capacitor from following a rapidly decreasing envelope and causes diagonal clipping.

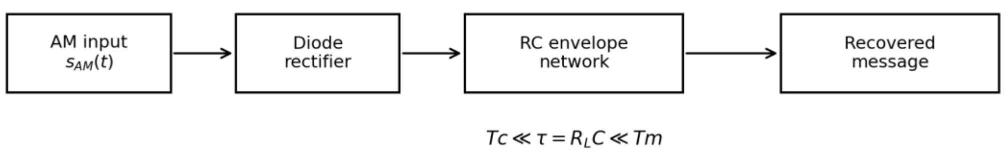

Fig. 5.3. Functional representation of an AM envelope detector and the time-constant condition.

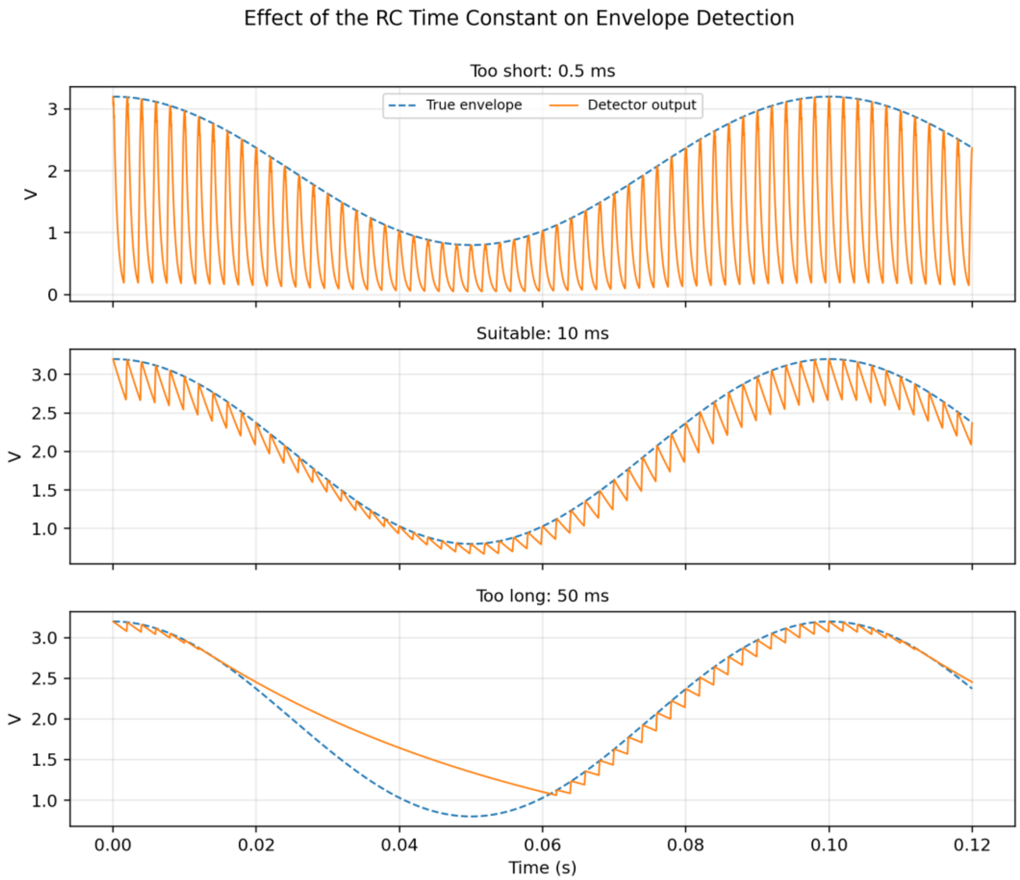

Fig. 5.4. Typical detector behavior for short, suitable, and long RC time constants.

2.5. Spectral Interpretation and Performance Measures

Before demodulation, the useful information is located in the sidebands around the carrier. Rectification and nonlinear peak detection create a low-frequency envelope component as well as residual carrier and harmonic components. Removing the DC term from the detector output and applying suitable low-pass smoothing yields an estimate of the message.

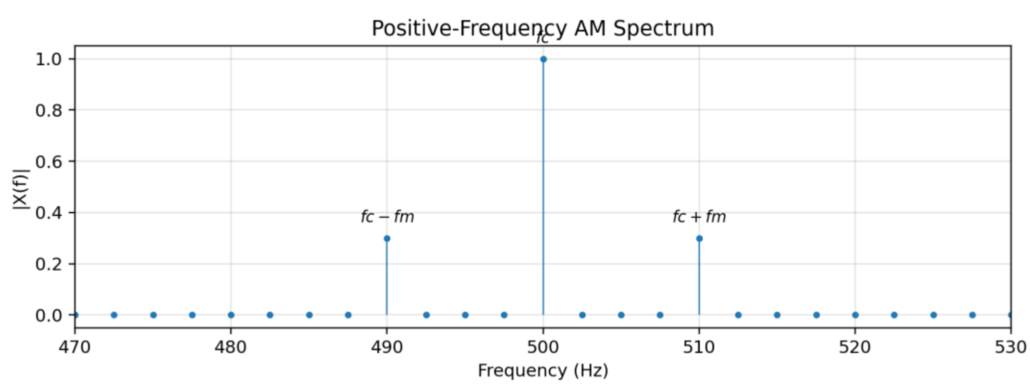

Fig. 5.5. Carrier and sideband components of the single-tone AM signal used in this experiment.

The agreement between the normalized original and recovered messages may be evaluated using the mean-square error and correlation coefficient:

$$MSE = (1/N)\Sigma[n=0 \text{ to } N\text{-}1](m[n]-\hat{m}[n])^2$$

$$\rho = corr(m[n],\hat{m}[n])$$

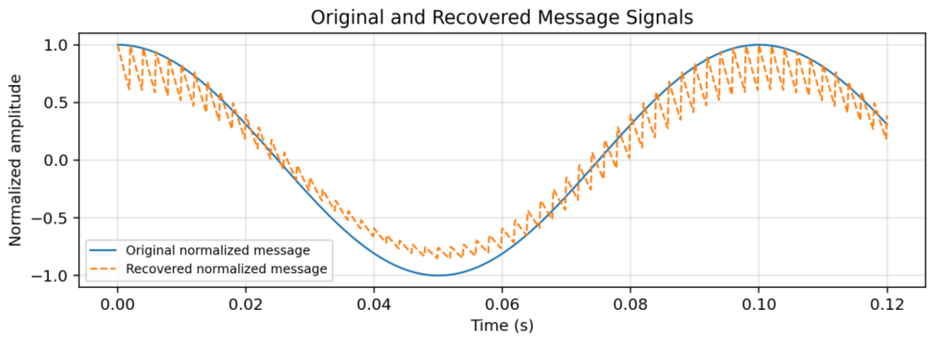

Fig. 5.6. Example comparison of the normalized original and recovered message signals.

3. Materials Used

- Computer
- MATLAB software
- Signal Processing Toolbox or an equivalent analytic-signal implementation
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands and programming structures used in this experiment are:

- cos, abs, max, mean, exp, fft, fftshift, fftfreq-equivalent axis construction
- hilbert
- plot, subplot, grid, xlabel, ylabel, title, legend
- for loops and logical comparisons for the diode-RC peak detector
- corrcoef or corr for waveform similarity

4. Preliminary (PreLab)

Complete the following calculations before attending the laboratory.

4.1. AM Signal Parameters

The parameters selected for the experiment are

$$Ac = 2\ V, \quad fm = 10\ Hz, \quad fc = 500\ Hz, \quad \mu = 0.6, \quad Fs = 50\ kHz$$

1. Calculate the lower- and upper-sideband frequencies.
2. Calculate the AM transmission bandwidth.
3. Calculate the expected amplitude of each sideband in the expanded time-domain expression.
4. Calculate Emax and Emin.
5. Use Emax and Emin to verify the modulation index.
6. Calculate the carrier period Tc and message period Tm.

Parameter	Expression	Calculated value
Lower sideband	fc - fm	
Carrier	fc	
Upper sideband	fc + fm	
AM bandwidth	2fm	
Sideband amplitude	μAc/2	
Maximum envelope	Ac(1+μ)	
Minimum envelope	Ac(1-μ)	

Parameter	Expression	Calculated value
Carrier period	1/fc	
Message period	1/fm	

4.2. Detector Time Constants

Classify each proposed time constant by comparing it with Tc and Tm. Predict the dominant distortion mechanism before running MATLAB.

Case	Time constant τ	Comparison with Tc and Tm	Predicted result
A	0.5 ms		
B	10 ms		
C	50 ms		

4.3. Modulation Index Cases

For μ=0.5, μ=1.0, and μ=1.3, calculate Emax and Emin. State whether ideal envelope detection is theoretically possible without distortion.

μ	Emax	Emin	Under / critical / overmodulated	Expected detector performance
0.5				
1.0				
1.3				

5. Procedure

Part 1: Generation and Spectral Analysis of a Conventional AM Signal

1. Open MATLAB and create a new script file.
2. Set Fs=50 kHz and define an observation interval of 0.4 s.
3. Generate m(t)=cos(2π10t), c(t)=2cos(2π500t), and s_AM(t)=2[1+0.6m(t)]cos(2π500t).
4. Plot the message, carrier, and AM waveform. Display only a short time interval for the carrier and AM plots.
5. Plot the theoretical upper and lower envelopes ±2[1+0.6m(t)] on the AM waveform.
6. Calculate the centered FFT of the AM signal and identify the components at fc-fm, fc, and fc+fm.
7. Record the measured frequencies and amplitudes in Table 5.3.
8. Calculate Emax, Emin, and the measured modulation index from the generated waveform.

```
Fs = 50000;
Ts = 1/Fs;
t = 0:Ts:0.4-Ts;
Ac = 2; fm = 10; fc = 500; mu = 0.6;
m = cos(2*pi*fm*t);
sAM = Ac*(1 + mu*m).*cos(2*pi*fc*t);
envTheory = Ac*(1 + mu*m);

N = length(sAM);
S = fftshift(fft(sAM))/N;
f = (-N/2:N/2-1)*(Fs/N);
```

Part 2: Ideal Envelope Extraction

1. Use the analytic signal to calculate the ideal envelope E_ideal(t)=|hilbert(s_AM(t))|.
2. Plot the theoretical and ideal MATLAB envelopes on the same axes.

3. Remove the DC term from the ideal envelope using mean subtraction.
4. Normalize the recovered waveform and compare it with the normalized original message.
5. Calculate the mean-square error and correlation coefficient.
6. Record the results in Tables 5.4 and 5.6.

```
envIdeal = abs(hilbert(sAM));
recIdeal = envIdeal - mean(envIdeal);
recIdeal = recIdeal/max(abs(recIdeal));
mNorm = m/max(abs(m));
mseIdeal = mean((mNorm-recIdeal).^2);
R = corrcoef(mNorm,recIdeal);
rhoIdeal = R(1,2);
```

Part 3: Practical Diode-RC Peak Detector

1. Half-wave rectify the AM signal using x_rect(t)=max[s_AM(t),0].
2. Implement the charge-and-discharge algorithm below for τ=10 ms.
3. When the rectified input is greater than the previous capacitor voltage, assign the input value to the detector output.
4. Otherwise, discharge the previous output exponentially using α=exp(-T_s/τ).
5. Plot the detector output and theoretical envelope.
6. Remove the DC component, normalize the recovered signal, and compare it with the original message.
7. Calculate MSE and correlation and enter the results in Table 5.6.

```
tau = 10e-3;
alpha = exp(-Ts/tau);
xrect = max(sAM,0);
envRC = zeros(size(xrect));
envRC(1) = xrect(1);
for n = 2:length(xrect)
    if xrect(n) >= envRC(n-1)
        envRC(n) = xrect(n);
    else
        envRC(n) = alpha*envRC(n-1);
    end
end
recRC = envRC - mean(envRC);
recRC = recRC/max(abs(recRC));
```

Part 4: Effect of the RC Time Constant

1. Repeat the practical detector for τ=0.5 ms, 10 ms, and 50 ms.
2. Plot the theoretical envelope and all detector outputs using separate subplots.
3. Observe the carrier ripple for the short time constant.
4. Observe whether the long time constant follows the decreasing parts of the envelope.
5. For every case, calculate MSE and correlation after DC removal and normalization.
6. Classify each waveform as ripple-dominated, acceptable, or diagonally clipped and complete Table 5.4.

Part 5: Effect of the Modulation Index

1. Keep Ac=2 V, fm=10 Hz, fc=500 Hz, and τ=10 ms.
2. Generate AM signals for μ=0.5, μ=1.0, and μ=1.3.
3. For each case, calculate the mathematical envelope, ideal envelope, and practical RC-detector output.
4. Plot the results using three subplots.
5. Calculate the modulation index from Emax and Emin whenever Emin is nonnegative.
6. Explain why the detector output for μ=1.3 does not reproduce the signed message waveform.
7. Complete Table 5.5.

Part 6: Frequency-Domain Verification After Detection

1. Calculate the FFT of the raw RC-detector output.
2. Identify the DC component, the recovered message component near 10 Hz, and any remaining carrier-related components.
3. Calculate the FFT after DC removal and normalization.
4. Compare the message peak before and after detection and record the results in Table 5.6.
5. Save the MATLAB script and all figures using the naming convention specified by the laboratory supervisor.

6. Evaluation of Results

Tabulate and interpret the data obtained during the experiment.

Table 5.3. Theoretical and MATLAB spectral components of the AM signal.

Component	Theoretical frequency (Hz)	FFT frequency (Hz)	Theoretical amplitude	FFT amplitude	Error (%)
Lower sideband					
Carrier					
Upper sideband					

Table 5.4. Envelope-detector time-constant comparison.

τ	Ripple amplitude	MSE	Correlation ρ	Dominant distortion	Assessment
0.5 ms					
10 ms					
50 ms					

Table 5.5. Effect of modulation index on envelope detection.

μ	Measured Emax	Measured Emin	Measured μ	Detector MSE	Observation
0.5					
1.0					
1.3					

Table 5.6. Demodulation performance summary.

Method / case	Message peak (Hz)	MSE	Correlation ρ	Residual carrier	Overall interpretation
Ideal envelope					
RC detector, τ=10 ms					
RC detector, τ=0.5 ms					
RC detector, τ=50 ms					

Answer the following questions:

1. Why does a conventional AM signal contain a carrier component and two sidebands?
2. How is the AM bandwidth related to the highest message frequency?
3. Why is the condition $\mu \leq 1$ required for undistorted envelope detection?
4. What physical operation does the diode perform in an envelope detector?
5. Why must the capacitor charge rapidly but discharge more slowly?
6. What type of distortion occurs when the RC time constant is too short?
7. What type of distortion occurs when the RC time constant is too long?
8. Why can the ideal Hilbert-transform envelope be used as a reference result?

9. Why does the practical detector output contain a DC component?
10. Which frequency components remain in the raw detector-output spectrum?
11. Explain why the mathematical envelope becomes negative during overmodulation while the detected envelope cannot follow the sign change.
12. Compare the ideal and practical detector results and discuss the main sources of error.
13. Based on the three tested values, recommend an appropriate range for τ relative to T_c and T_m.
14. How would increasing the message frequency affect the optimum RC time constant?

Experiment 6: DSB-SC Modulation and Coherent Detection

1. Objective of the Experiment

The objective of this experiment is to investigate double-sideband suppressed-carrier (DSB-SC) modulation and coherent detection using MATLAB. Students will generate a baseband message, translate its spectrum around a carrier frequency through multiplication, and verify that the resulting signal contains upper and lower sidebands while the discrete carrier component is suppressed.

The experiment also examines balanced modulation, synchronous product detection, low-pass filtering, and the effect of carrier synchronization errors. By changing the phase and frequency of the receiver local oscillator, students will observe why coherent demodulation requires a locally generated carrier that is synchronized with the received signal.

At the end of the experiment, students are expected to:

- Generate and analyze a multi-tone DSB-SC signal in the time and frequency domains.
- Identify all upper- and lower-sideband components and calculate the occupied bandwidth.
- Explain why the carrier line is absent from the ideal DSB-SC spectrum.
- Implement a balanced modulator by subtracting two conventional AM signals.
- Recover the message using a product detector and a low-pass filter.
- Quantify the effect of local-oscillator phase error on the recovered signal amplitude.
- Observe the time-varying distortion caused by local-oscillator frequency error.
- Compare theoretical predictions with MATLAB results using frequency, gain, MSE, and correlation measurements.

2. Theoretical Background

2.1. Double-Sideband Suppressed-Carrier Modulation

In DSB-SC modulation, a baseband message signal is multiplied directly by a sinusoidal carrier. The modulated signal is

$$s\_DSB\text{-}SC(t) = Ac\, m(t) \cos(2\pi fct)$$

where Ac is the carrier amplitude, m(t) is the message, and fc is the carrier frequency. Multiplication by the carrier shifts the message spectrum to the positive and negative carrier frequencies:

$$S\_DSB\text{-}SC(f) = (Ac/2)[M(f\text{-}fc) + M(f\text{+}fc)]$$

Unlike conventional AM, an additional carrier term Ac cos(2πfct) is not transmitted. For a message bandwidth W, each translated copy occupies W hertz on either side of the carrier, and the transmission bandwidth is

$$BW\_DSB\text{-}SC = 2W$$

For a single-tone message m(t)=Am cos(2πfmt), the product identity gives

$$s\_DSB\text{-}SC(t) = (AcAm/2)\{\cos[2\pi(fc\text{-}fm)t] + \cos[2\pi(fc\text{+}fm)t]\}$$

Therefore, only the lower and upper sidebands occur at fc-fm and fc+fm. The absence of a carrier component improves power efficiency, but it prevents the use of a simple envelope detector.

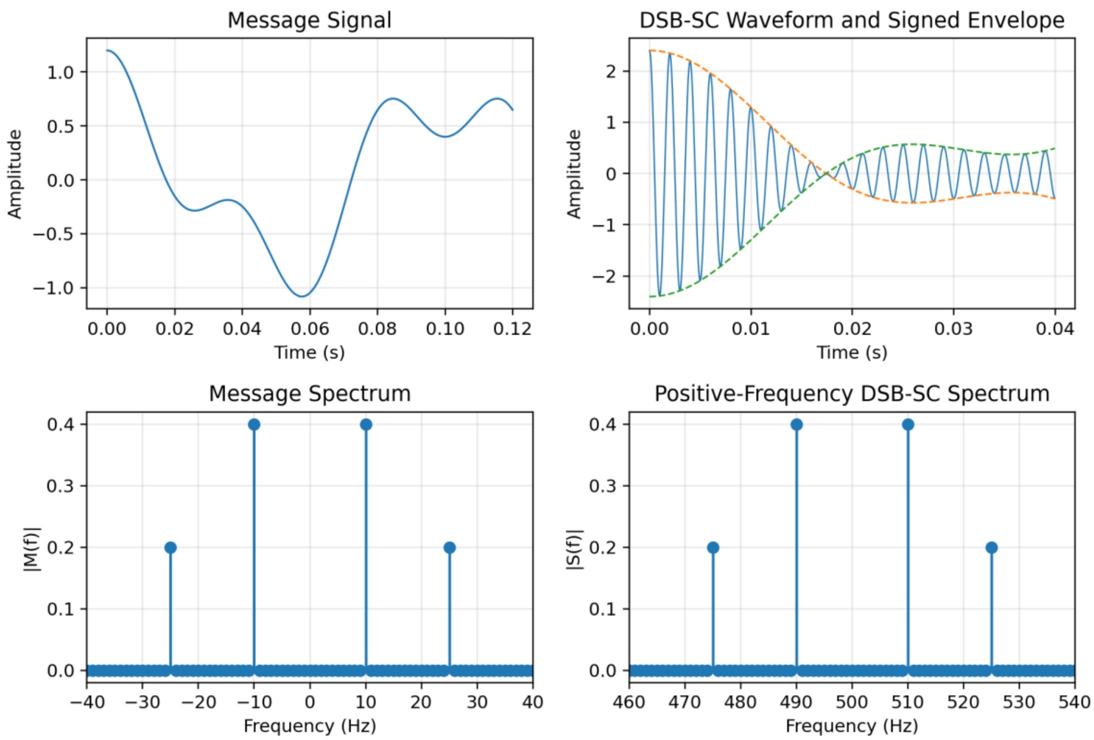

Fig. 6.1. Time- and frequency-domain representation of a multi-tone DSB-SC signal.

2.2. Balanced Modulation

The course notes present balanced modulation as a practical method of cancelling the carrier. Two conventional AM signals are generated using message inputs of opposite polarity:

$$s1(t) = Ac[1 + kam(t)]cos(2\pi fct)$$

$$s2(t) = Ac[1 - kam(t)]cos(2\pi fct)$$

Subtracting the second signal from the first cancels the equal carrier terms and retains the message-carrier product:

$$s1(t) - s2(t) = 2Ackam(t)cos(2\pi fct)$$

The scaling factor depends on Ac and ka, but the spectral form is DSB-SC. Any mismatch between the two branches may leave a residual carrier component.

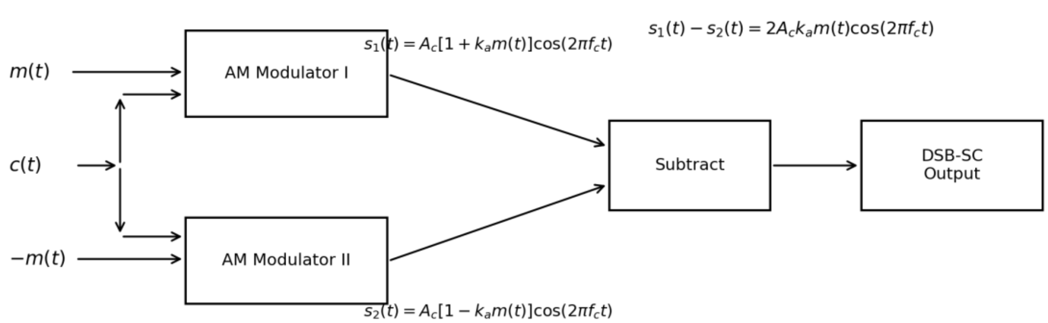

Fig. 6.2. Balanced-modulator principle based on subtracting two AM signals.

2.3. Coherent Product Detection

A DSB-SC signal is demodulated by multiplying it with a locally generated sinusoid and then applying a low-pass filter. Let the local oscillator be

$$cLO(t) = 2\cos(2\pi fLOt + \varphi)$$

For ideal synchronization, fLO=fc and φ=0. The product-detector output before filtering is

$$v(t) = 2Ac\, m(t)\cos(2\pi fct)\cos(2\pi fct)$$

$$v(t) = Ac\, m(t) + Ac\, m(t)\cos(4\pi fct)$$

The low-pass filter removes the translated component around 2fc and retains the baseband term Ac m(t). Dividing by Ac recovers the original message amplitude.

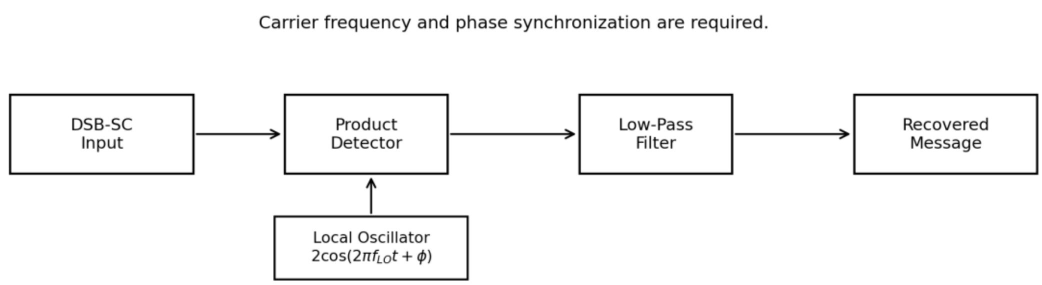

Fig. 6.3. Coherent product detector with a synchronized local oscillator.

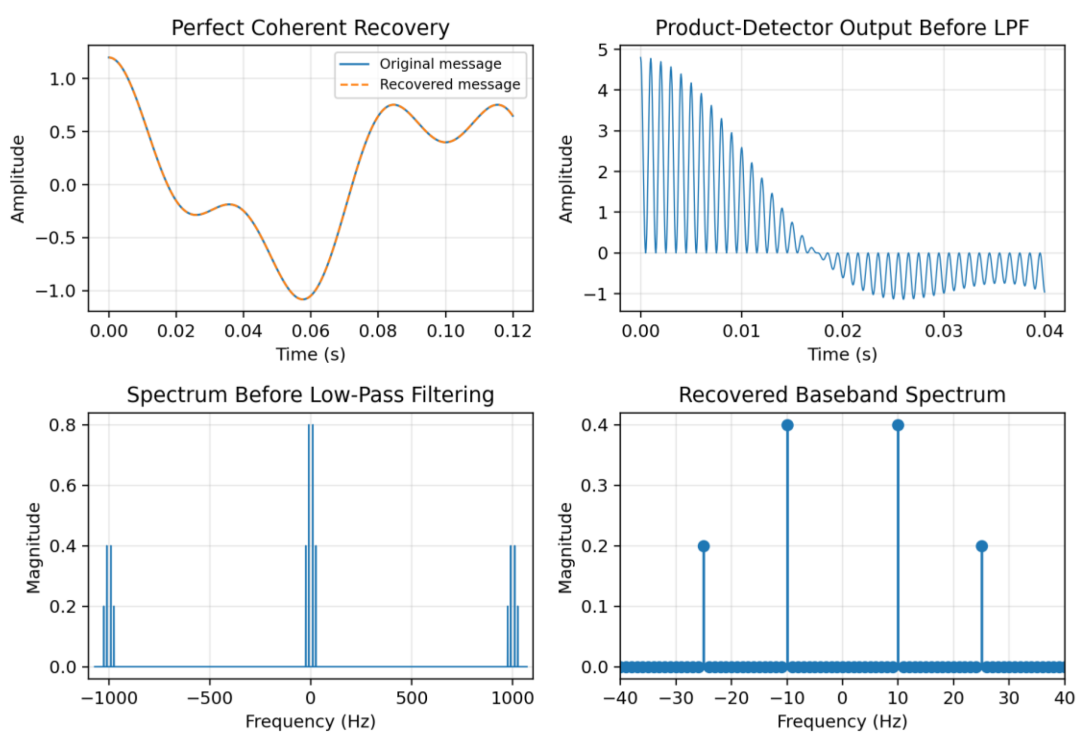

Fig. 6.4. Perfect coherent recovery and the role of low-pass filtering.

2.4. Effect of Phase Error

When the local oscillator has the correct frequency but a phase difference φ, the low-pass output becomes

$$vLPF(t) = Ac\, m(t)\cos(\varphi)$$

The recovered waveform retains its shape but is scaled by cos(φ). At φ=90°, the desired output ideally becomes zero. At φ=180°, the recovered message is inverted. This phase sensitivity distinguishes coherent detection from envelope detection.

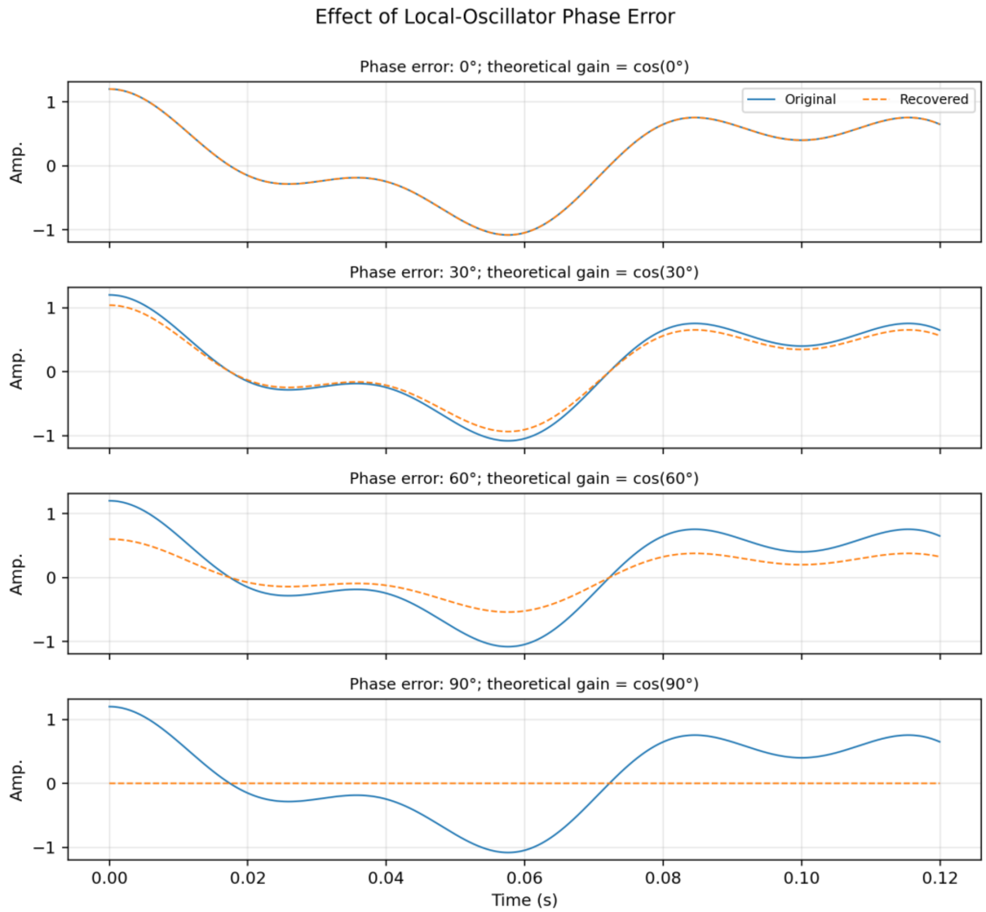

Fig. 6.5. Recovered message for selected local-oscillator phase errors.

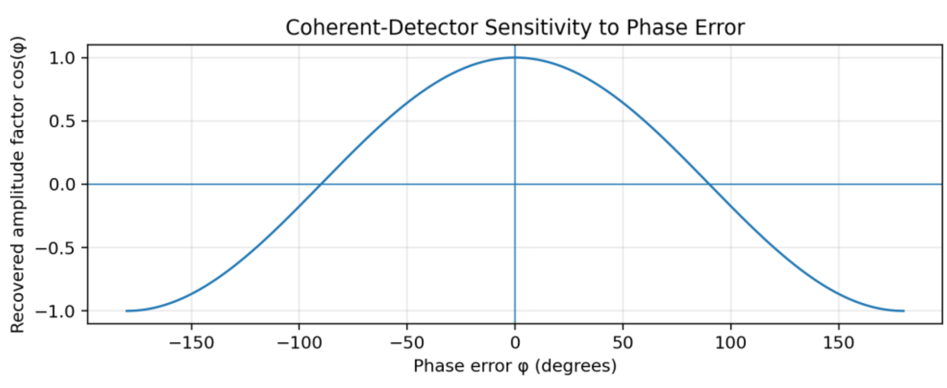

Fig. 6.6. Theoretical coherent-detector amplitude factor as a function of phase error.

2.5. Effect of Frequency Error

If the receiver oscillator frequency is fLO=fc+Δf, multiplication followed by low-pass filtering yields

$$vLPF(t) = Ac\, m(t)\, cos[2\pi\Delta f\, t + \varphi]$$

The recovered message is multiplied by a slowly varying beat term. Thus, even a small carrier-frequency mismatch creates time-varying amplitude and sign changes. In the frequency domain, each baseband component is shifted by ±Δf rather than returning exactly to its original frequency.

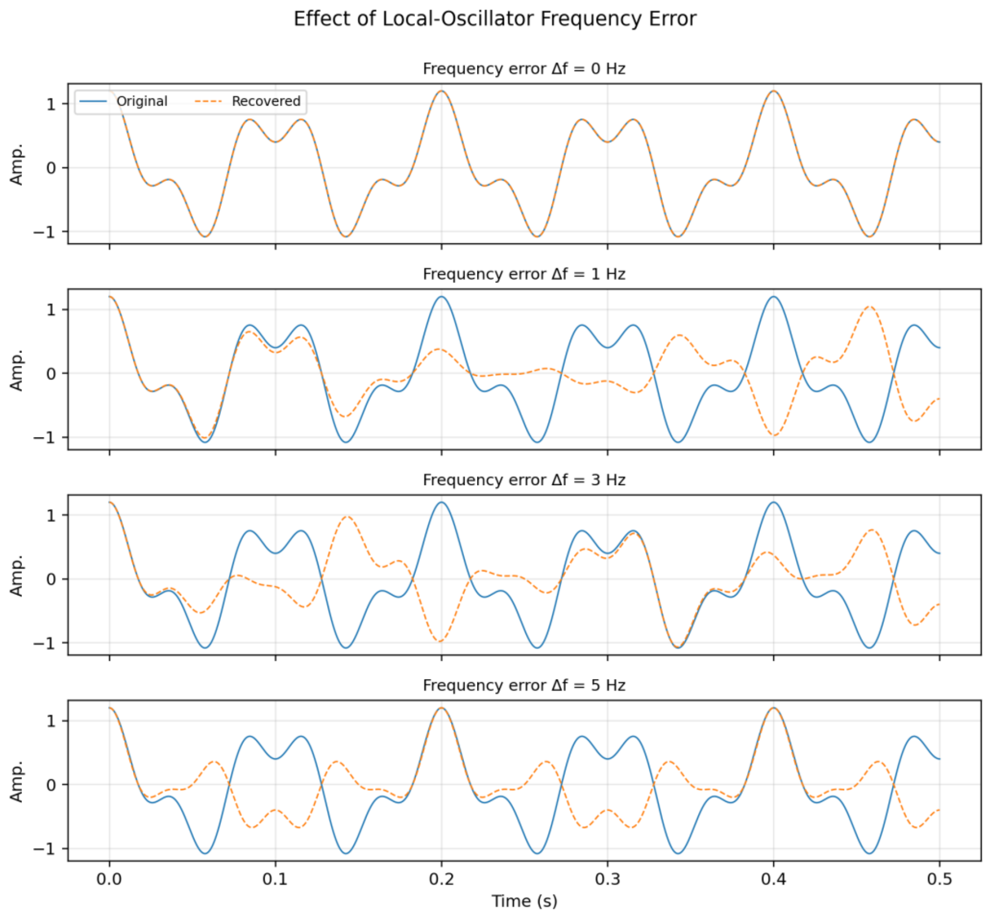

Fig. 6.7. Time-varying distortion caused by local-oscillator frequency mismatch.

2.6. Carrier Recovery and the Costas Receiver

Because the transmitted DSB-SC signal does not contain a discrete carrier line, the receiver must regenerate a synchronized carrier. The course notes introduce the Costas receiver, which uses in-phase and quadrature product-detector branches, low-pass filters, a phase-error detector, and a voltage-controlled oscillator. The feedback loop adjusts the local oscillator until the quadrature error approaches zero. This experiment does not implement a complete feedback loop, but the phase- and frequency-error studies demonstrate why carrier recovery is necessary.

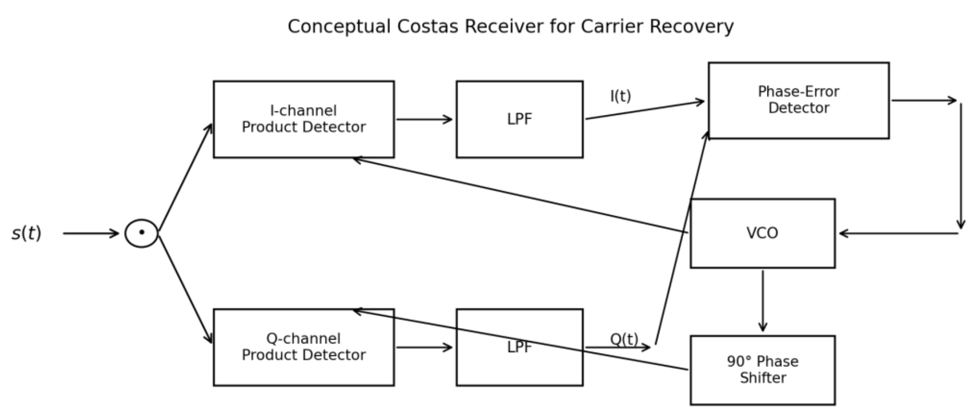

Fig. 6.8. Conceptual Costas receiver used for DSB-SC carrier recovery.

3. Materials Used

- Computer
- MATLAB software
- Signal Processing Toolbox or an equivalent low-pass-filter implementation
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands and operations used in this experiment are:

- cos, deg2rad, abs, max, mean, fft, fftshift, length
- lowpass or an equivalent FIR/IIR low-pass filter
- plot, stem, subplot, grid, xlabel, ylabel, title, legend
- corrcoef and mean-square-error calculations
- element-wise multiplication and signal subtraction

4. Preliminary (PreLab)

Complete the following calculations before attending the laboratory.

4.1. Message and DSB-SC Spectrum

The signals and parameters selected for the experiment are

$$m(t) = 0.8\cos(2\pi 10t) + 0.4\cos(2\pi 25t)$$

$$s(t) = 2m(t)\cos(2\pi 500t), \quad Fs = 20\ kHz$$

1. Determine the highest message frequency W.
2. Calculate the DSB-SC transmission bandwidth.
3. List all positive-frequency lower- and upper-sideband components.
4. Calculate the theoretical amplitude of each positive-frequency spectral line in the normalized two-sided FFT representation.
5. Sketch the theoretical message spectrum and positive-frequency DSB-SC spectrum.
6. Explain why no spectral line is expected at 500 Hz.

Message component	Message frequency (Hz)	Lower sideband (Hz)	Upper sideband (Hz)	Expected line amplitude
First tone				
Second tone				

4.2. Balanced-Modulator Derivation

1. Expand s1(t)=Ac[1+kam(t)]cos(2πfct).
2. Expand s2(t)=Ac[1-kam(t)]cos(2πfct).
3. Derive s1(t)-s2(t) and identify the value of ka required for equality with 2m(t)cos(2π500t) when Ac=2.
4. Predict the residual carrier if the carrier amplitudes in the two branches differ by 2 percent.

4.3. Coherent-Detector Predictions

1. Derive the product-detector output for perfect synchronization.
2. Identify the baseband and high-frequency terms before low-pass filtering.
3. Calculate the theoretical recovered amplitude factor cos(φ) for φ=0°, 30°, 60°, 90°, and 180°.
4. Write the recovered signal expression for Δf=3 Hz and φ=0°.
5. Predict whether an envelope detector can correctly recover this DSB-SC waveform and justify your answer.

Phase error φ	Theoretical gain cos(φ)	Expected polarity	Expected correlation sign
0°			
30°			
60°			
90°			
180°			

5. Procedure

Part 1: Generation and Spectral Analysis of the DSB-SC Signal

1. Open MATLAB and create a new script file.
2. Set Fs=20 kHz and define a one-second observation interval.
3. Generate m(t)=0.8cos(2π10t)+0.4cos(2π25t).
4. Generate the carrier c(t)=cos(2π500t) and DSB-SC signal s(t)=2m(t)c(t).
5. Plot the message and a short section of the modulated waveform.
6. Overlay the signed curves ±2m(t) on the DSB-SC waveform and interpret the phase reversals.
7. Calculate centered, normalized FFTs of m(t) and s(t).
8. Identify the spectral components and enter the results in Table 6.2.
9. Confirm that no discrete carrier component appears at 500 Hz.

```
Fs = 20000;
Ts = 1/Fs;
t = 0:Ts:1-Ts;
Ac = 2; fc = 500;
m = 0.8*cos(2*pi*10*t) + 0.4*cos(2*pi*25*t);
c = cos(2*pi*fc*t);
sDSB = Ac*m.*c;

N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
M = fftshift(fft(m))/N;
S = fftshift(fft(sDSB))/N;
```

Part 2: Balanced-Modulator Verification

1. Set ka=0.5 and generate s1(t)=2[1+kam(t)]cos(2π500t).
2. Generate s2(t)=2[1-kam(t)]cos(2π500t).
3. Calculate sBM(t)=s1(t)-s2(t).
4. Compare sBM(t) with the directly generated DSB-SC signal.
5. Calculate the maximum absolute difference and the MSE between the two signals.
6. Compare their spectra and verify carrier cancellation.
7. Introduce a 2 percent amplitude mismatch in the second branch and measure the residual carrier at 500 Hz.

```
ka = 0.5;
s1 = Ac*(1 + ka*m).*cos(2*pi*fc*t);
s2 = Ac*(1 - ka*m).*cos(2*pi*fc*t);
sBM = s1 - s2;
maxDifference = max(abs(sBM-sDSB));
mseBalanced = mean((sBM-sDSB).^2);
```

Part 3: Perfect Coherent Detection

1. Generate a synchronized local oscillator cLO(t)=2cos(2π500t).
2. Multiply the received DSB-SC signal by the local oscillator.
3. Plot the product-detector output in the time domain.

4. Calculate its spectrum and identify the baseband terms and components around 2fc.
5. Apply a low-pass filter with a cutoff frequency of approximately 60 Hz.
6. Divide the filtered output by Ac to obtain the recovered message.
7. Plot the original and recovered messages on the same axes.
8. Calculate MSE, correlation, and recovered amplitudes at 10 Hz and 25 Hz.
9. Complete Table 6.4.

```
cLO = 2*cos(2*pi*fc*t);
vProduct = sDSB.*cLO;
vLPF = lowpass(vProduct,60,Fs);
mRecovered = vLPF/Ac;
msePerfect = mean((m-mRecovered).^2);
R = corrcoef(m,mRecovered);
rhoPerfect = R(1,2);
```

Part 4: Effect of Local-Oscillator Phase Error

1. Repeat coherent detection for φ=0°, 30°, 60°, 90°, and 180° while keeping fLO=fc.
2. For each case, calculate the recovered peak amplitude and the amplitude ratio relative to the perfect case.
3. Compare the measured ratio with cos(φ).
4. Calculate the MSE and correlation coefficient without independently rescaling each recovered waveform.
5. Observe the zero output near 90° and polarity inversion at 180°.
6. Enter the results in Table 6.5.

```
phaseList = [0 30 60 90 180];
for k = 1:length(phaseList)
    phi = deg2rad(phaseList(k));
    cLOphi = 2*cos(2*pi*fc*t + phi);
    v = sDSB.*cLOphi;
    recPhi{k} = lowpass(v,60,Fs)/Ac;
end
```

Part 5: Effect of Local-Oscillator Frequency Error

1. Repeat coherent detection for Δf=0, 1, 3, and 5 Hz with φ=0°.
2. Use fLO=fc+Δf for each case.
3. Plot the original and recovered signals over at least 0.5 seconds.
4. Observe the beat envelope and sign reversals caused by Δf.
5. Calculate the recovered spectrum and determine the locations of the shifted components.
6. Calculate MSE and correlation over the complete one-second record.
7. Enter the results in Table 6.6.

```
deltaF = [0 1 3 5];
for k = 1:length(deltaF)
    fLO = fc + deltaF(k);
    cLOf = 2*cos(2*pi*fLO*t);
    v = sDSB.*cLOf;
    recFreq{k} = lowpass(v,60,Fs)/Ac;
end
```

Part 6: Comparison and Interpretation

1. Compare the directly generated and balanced-modulator DSB-SC signals.
2. Compare the transmitted DSB-SC spectrum with the product-detector spectrum before filtering.
3. Explain which terms are removed by the low-pass filter.
4. Compare perfect synchronization, phase mismatch, and frequency mismatch.
5. Prepare all tables and answer the post-laboratory questions.
6. Save the MATLAB script and all figures using the file-naming convention specified by the laboratory supervisor.

6. Evaluation of Results

Tabulate and interpret the data obtained during the experiment.

Table 6.1. PreLab calculations for message and DSB-SC components.

Quantity	Theoretical result	MATLAB result	Error (%)
Message bandwidth W			
DSB-SC bandwidth 2W			
Carrier component at fc			
Highest positive-frequency component			
Lowest positive-frequency component			

Table 6.2. Theoretical and measured positive-frequency DSB-SC spectral lines.

Component	Theoretical frequency (Hz)	FFT frequency (Hz)	Theoretical amplitude	FFT amplitude	Error (%)
fc - 25 Hz					
fc - 10 Hz					
fc					
fc + 10 Hz					
fc + 25 Hz					

Table 6.3. Balanced-modulator verification.

Case	Maximum absolute difference	MSE	Carrier amplitude at 500 Hz	Interpretation
Ideal branch matching				
2% branch mismatch				

Table 6.4. Perfect coherent-detection results.

Quantity	Theoretical value	Measured value	Error (%)
Recovered 10 Hz amplitude			
Recovered 25 Hz amplitude			
Correlation coefficient			
MSE			
LPF cutoff frequency			

Table 6.5. Effect of local-oscillator phase error.

φ	Theoretical cos(φ)	Measured gain	MSE	Correlation ρ	Observation
0°					
30°					
60°					
90°					
180°					

Table 6.6. Effect of local-oscillator frequency error.

Δf (Hz)	Expected beat period 1/Δf	Measured behavior	MSE	Correlation ρ	Spectral observation
0	Not applicable				
1					
3					

Δf (Hz)	Expected beat period 1/Δf	Measured behavior	MSE	Correlation ρ	Spectral observation
5					

Answer the following questions:

1. Why is the carrier component absent from the ideal DSB-SC spectrum?
2. Why does DSB-SC retain the same bandwidth as conventional AM although the carrier is suppressed?
3. Explain the phase reversals visible in the DSB-SC time waveform when the message changes sign.
4. How does a balanced modulator cancel the carrier component?
5. What practical imperfection produces residual carrier in a balanced modulator?
6. Why cannot a conventional envelope detector recover a DSB-SC signal correctly?
7. Which spectral components are produced at the output of the coherent product detector before filtering?
8. Why is the local oscillator amplitude selected as 2 in the theoretical derivation?
9. How does the low-pass-filter cutoff frequency relate to the message bandwidth?
10. Why does a phase error scale the recovered signal by cos(φ)?
11. What happens to the recovered message at phase errors of 90° and 180°?
12. Why does frequency mismatch create a time-varying beat rather than only a constant gain error?
13. How are the recovered baseband spectral lines displaced when Δf is nonzero?
14. Compare phase error and frequency error in terms of their effects on message recovery.
15. Explain why a Costas receiver is useful for DSB-SC demodulation.
16. Compare conventional AM and DSB-SC in terms of receiver complexity and transmitted carrier power.

Experiment 7: Single-Sideband Modulation and Coherent Detection

1. Objective of the Experiment

The objective of this experiment is to investigate single-sideband (SSB) modulation and coherent detection using MATLAB. Students will begin with a double-sideband suppressed-carrier signal and generate upper-sideband (USB) and lower-sideband (LSB) signals by frequency discrimination. They will then generate the same signals by the phase-shift method using the Hilbert transform and compare the two implementations in the time and frequency domains.

The experiment also examines SSB bandwidth, sideband suppression, coherent message recovery, and the sensitivity of the phase-shift method to quadrature phase error. The MATLAB results will be compared with the frequency-domain relationships and Hartley-modulator structure presented in the course notes.

At the end of the experiment, students are expected to:

- Generate DSB-SC, USB, and LSB signals for a multi-tone message.
- Identify the frequencies retained in USB and LSB spectra.
- Explain why SSB requires only the message bandwidth W rather than 2W.
- Implement frequency-discrimination and Hilbert-transform methods of SSB generation.
- Use the MATLAB hilbert command to obtain the quadrature message signal.
- Compare USB and LSB signals produced by two different methods.
- Recover the message from either sideband by coherent detection and low-pass filtering.
- Calculate sideband suppression ratio and evaluate quadrature phase-shifter error.

2. Theoretical Background

2.1. Motivation for Single-Sideband Transmission

A DSB-SC signal contains two translated copies of the message spectrum. Since the upper and lower sidebands contain equivalent message information, transmitting both sidebands is not always necessary. In SSB modulation, one sideband and the carrier are suppressed. If the baseband message bandwidth is W, the DSB-SC bandwidth and SSB bandwidth are

$$BW\_DSB\text{-}SC = 2W$$

$$BW\_SSB = W$$

Suppressing one sideband reduces the required transmission bandwidth by one half relative to DSB-SC. The transmitted power is also concentrated in only one information-bearing sideband.

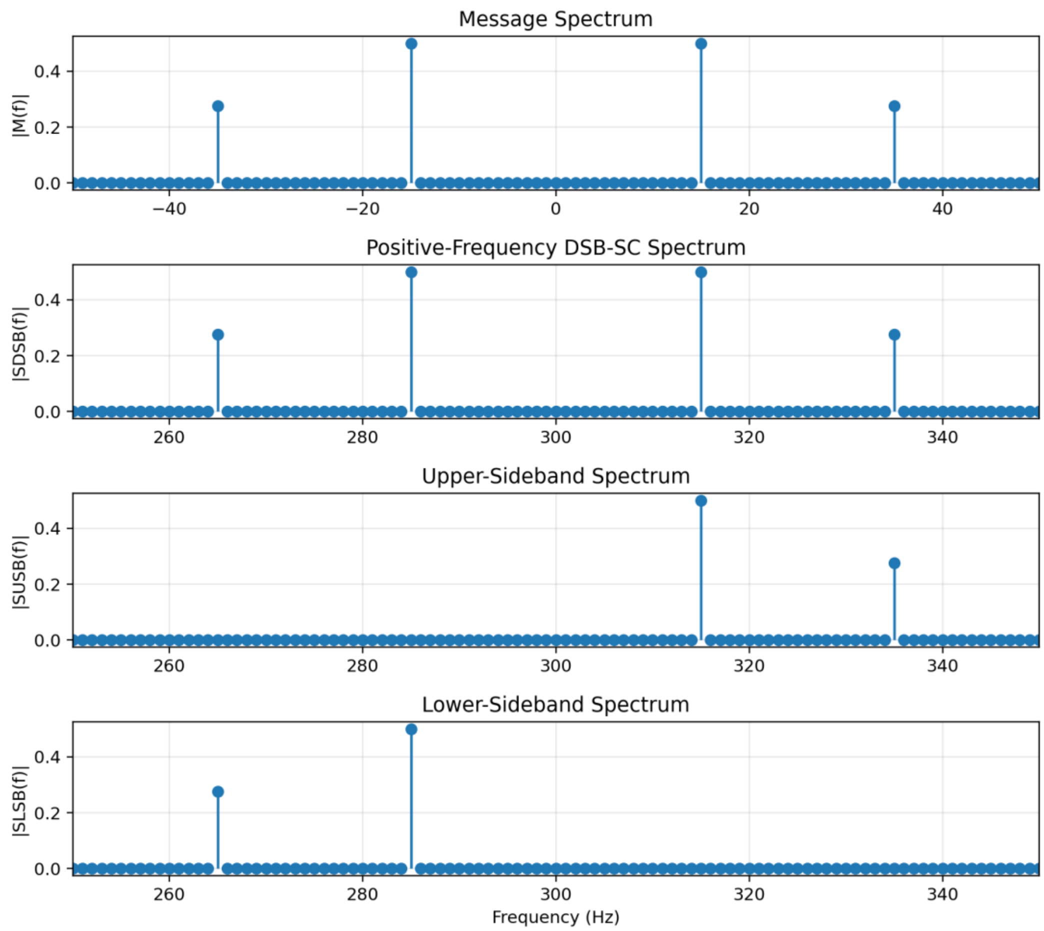

Fig. 7.1. Message, DSB-SC, USB, and LSB spectra for a two-tone message.

2.2. Frequency-Discrimination Method

In the frequency-discrimination method, the message is first multiplied by a carrier to produce a DSB-SC signal. A sideband-selecting band-pass filter then passes either the frequency region above the carrier for USB or the region below the carrier for LSB. For a message component at fm, the translated DSB-SC components occur at fc-fm and fc+fm. The USB retains fc+fm, whereas the LSB retains fc-fm.

The course notes emphasize the need for spectral separation and a realizable transition region. If the message contains significant energy very close to zero frequency, an ideal sharp sideband filter is difficult to realize because the desired and rejected sidebands meet at the carrier frequency. A guard band or an alternative phase-shift method may therefore be required.

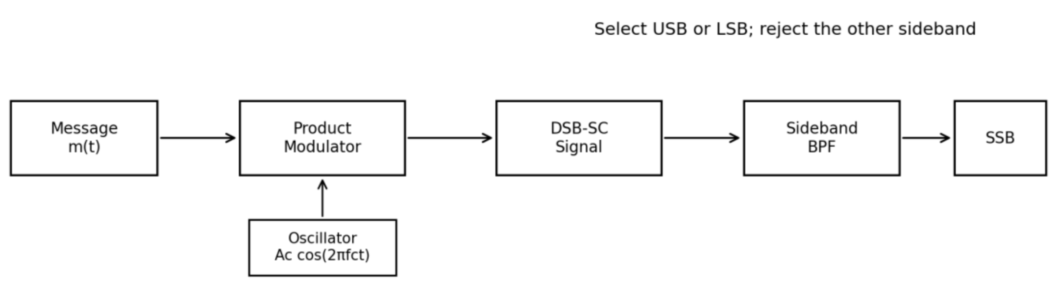

Fig. 7.2. SSB generation by frequency discrimination and sideband filtering.

2.3. Hilbert Transform

The Hilbert transform produces a quadrature version of a real signal. In the course notes, the ideal Hilbert-transform frequency response is written as

$$H(f) = -j\,\mathrm{sgn}(f)$$

Thus, positive-frequency components undergo a -90° phase shift and negative-frequency components undergo a +90° phase shift, while the magnitude spectrum is unchanged. If $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$, then $m(t)$ and $\hat{m}(t)$ have the same magnitude spectrum but differ in phase.

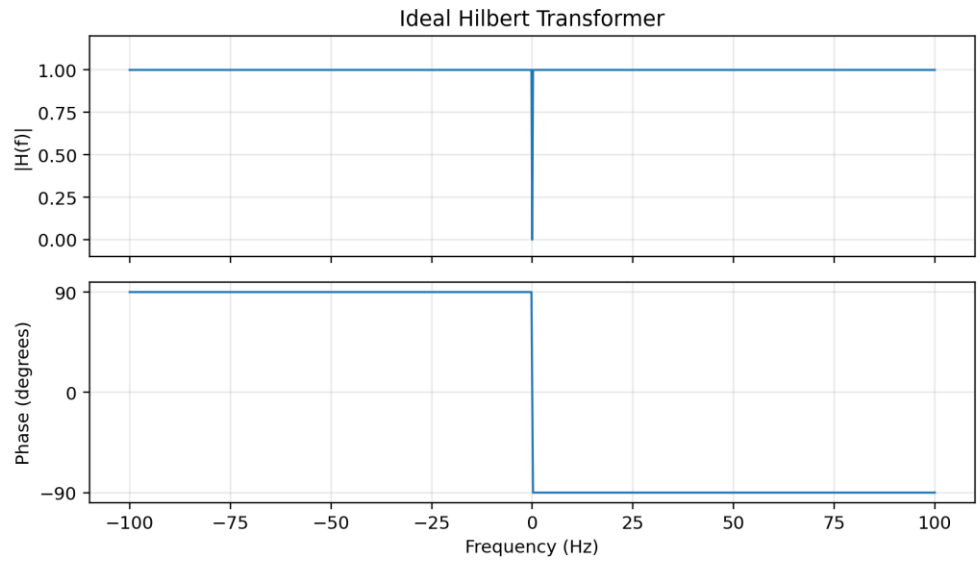

Fig. 7.3. Magnitude and phase response of the ideal Hilbert transformer.

2.4. Phase-Shift or Hartley Method

The Hartley modulator combines an in-phase product-modulator output with a quadrature product-modulator output. Using the sign convention in the course notes, the SSB signals are

$$s_{USB}(t) = (A_c/2)[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)]$$

$$s_{LSB}(t) = (A_c/2)[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)]$$

The subtraction or addition causes one translated sideband to cancel while the other sideband is reinforced. Unlike the frequency-discrimination method, sideband selection is achieved through phase relationships rather than a highly selective RF band-pass filter.

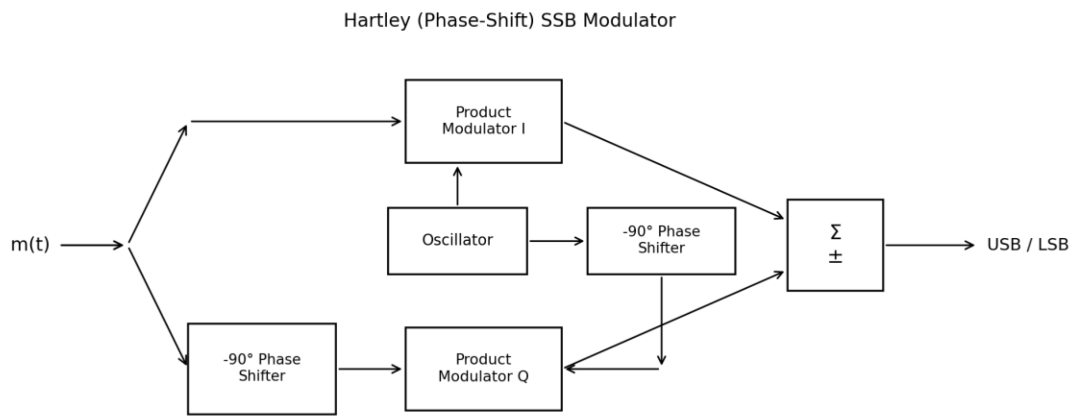

Fig. 7.4. Hartley phase-shift modulator for USB and LSB generation.

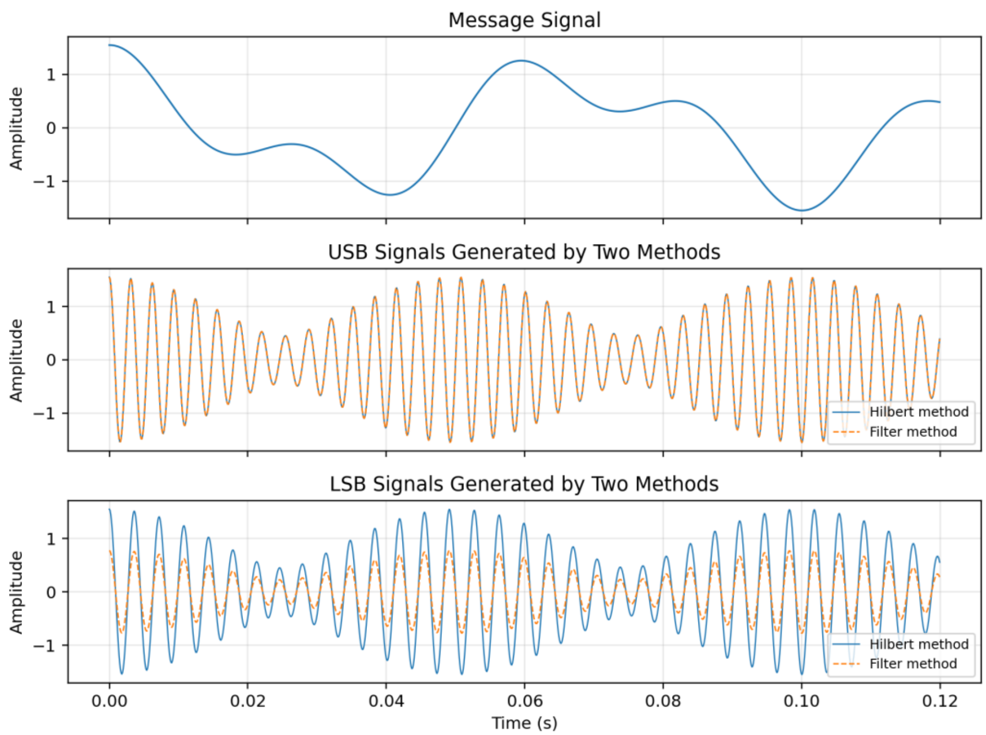

Fig. 7.5. Time-domain comparison of filter-method and Hilbert-method SSB signals.

2.5. Coherent Detection of SSB

An SSB signal can be demodulated by a product detector followed by a low-pass filter. When the local oscillator is synchronized with the transmitter carrier, multiplying either sideband by 2cos(2πfct) produces a baseband term proportional to m(t) and high-frequency components near 2fc. For the signal scaling used above, the low-pass component is

$$v\_LPF(t) = (Ac/2)m(t)$$

The original message is recovered by dividing the filtered signal by Ac/2. As in DSB-SC coherent detection, carrier-frequency and phase synchronization remain important.

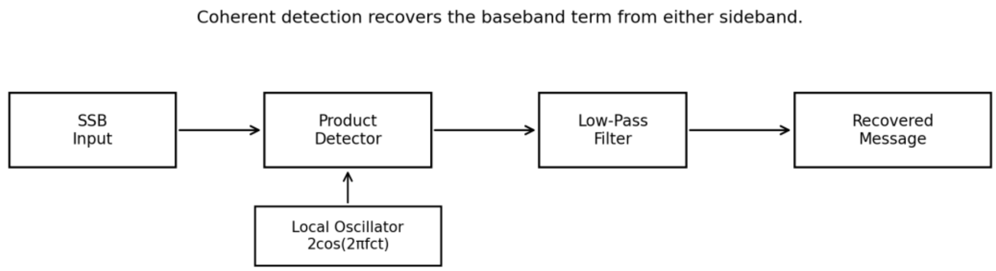

Fig. 7.6. Coherent SSB detector with product detection and low-pass filtering.

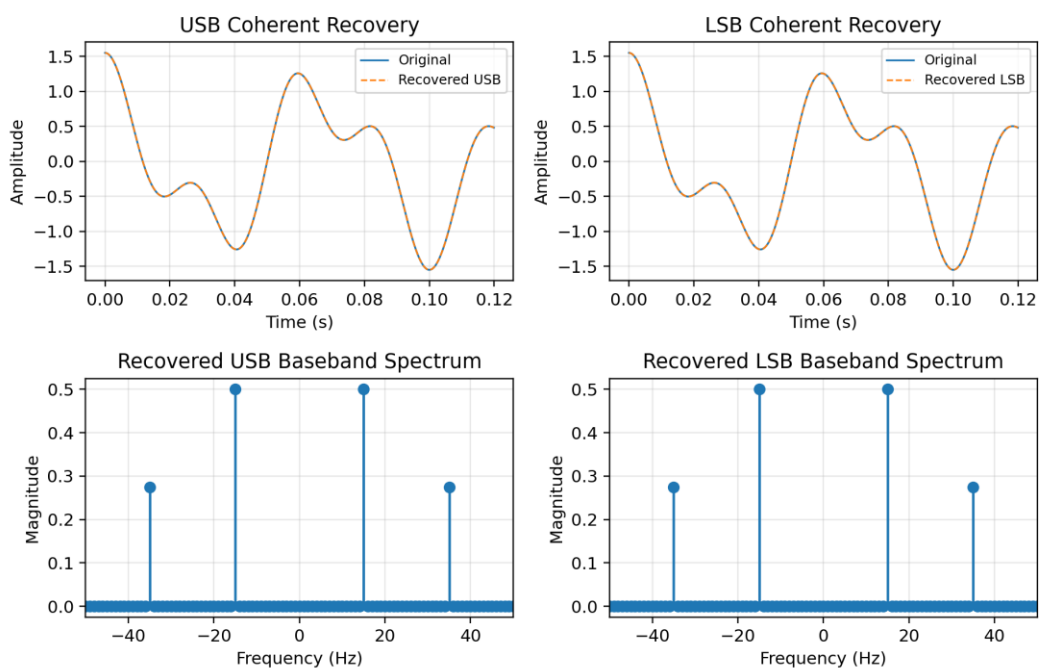

Fig. 7.7. Coherent recovery of the message from USB and LSB signals.

2.6. Sideband Suppression Ratio and Phase Error

Ideal 90° phase shifts cause complete cancellation of the unwanted sideband. A phase error in either quadrature path prevents exact cancellation and produces residual energy in the suppressed sideband. The sideband suppression ratio may be calculated from the desired and undesired spectral amplitudes as

$$SSR\_dB = 20 \log10(Adesired / Aundesired)$$

A larger SSR indicates better suppression. The ideal mathematical result is unbounded because the undesired sideband amplitude is zero; numerical and practical systems always produce a finite value due to sampling, leakage, amplitude mismatch, and phase error.

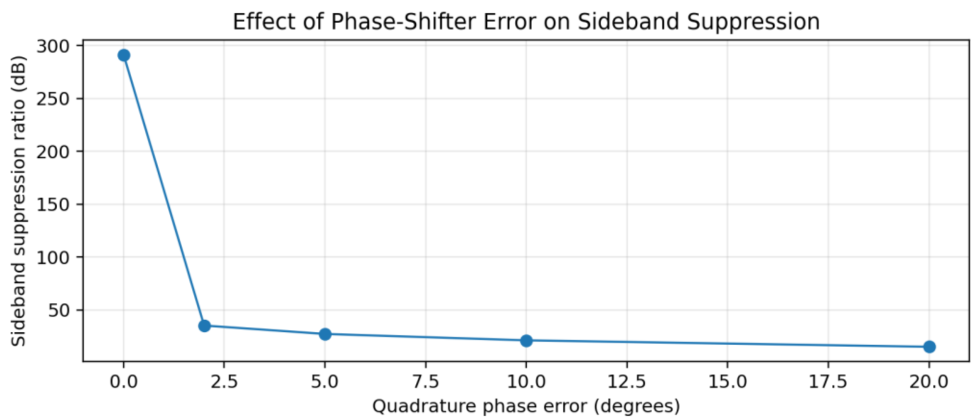

Fig. 7.8. Example relationship between quadrature phase error and sideband suppression ratio.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Signal Processing Toolbox, including the hilbert and lowpass functions
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, sin, hilbert, real, imag
- fft, fftshift, ifft, abs
- lowpass or an equivalent FFT-domain low-pass filter
- plot, stem, subplot, grid, legend
- mean, max, corrcoef, angle, and unwrap

4. Preliminary (PreLab)

Use the following parameters throughout the preliminary calculations unless otherwise stated:

$$m(t) = \cos(2\pi 15t) + 0.55\cos(2\pi 35t)$$

$$Ac = 2, \quad fc = 300\ Hz, \quad Fs = 10\ kHz$$

Complete the following tasks before attending the laboratory:

1. Determine the message bandwidth W.
2. Calculate the DSB-SC and SSB transmission bandwidths.
3. List all positive-frequency DSB-SC components.
4. List the positive-frequency components retained in the USB signal.
5. List the positive-frequency components retained in the LSB signal.
6. Write the Hilbert transform of each cosine component in m(t).
7. Using the course-note equations, write the complete time-domain USB and LSB expressions.
8. Derive the low-frequency component obtained after multiplying an SSB signal by $2\cos(2\pi fct)$.
9. Explain why an ideal filter must change from passband to stopband at the carrier frequency when the message contains components arbitrarily close to DC.
10. Predict how a quadrature phase error will affect the undesired sideband.

Table 7.1. Preliminary spectral calculations.

Quantity	Theoretical result	Unit / comment
Message bandwidth W		
DSB-SC bandwidth		
SSB bandwidth		
USB components		
LSB components		
Carrier component		
Expected LPF output scaling		

5. Procedure

Part 1: Message and DSB-SC Generation

1. Open MATLAB and create a new script file.
2. Set Fs=10 kHz and define a one-second observation interval.
3. Generate the two-tone message $m(t)=\cos(2\pi 15t)+0.55\cos(2\pi 35t)$.
4. Generate the carrier $\cos(2\pi 300t)$ and the DSB-SC signal $sDSB(t)=2m(t)\cos(2\pi 300t)$.
5. Plot the message and DSB-SC signals in the time domain.
6. Calculate centered and normalized FFTs.
7. Identify all positive-frequency DSB-SC components and complete the relevant result table.

```
Fs = 10000;
t = 0:1/Fs:1-1/Fs;
Ac = 2; fc = 300;
m = cos(2*pi*15*t) + 0.55*cos(2*pi*35*t);
sDSB = Ac*m.*cos(2*pi*fc*t);
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
M = fftshift(fft(m))/N;
SDSB = fftshift(fft(sDSB))/N;
```

Part 2: SSB Generation by Frequency Discrimination

1. Obtain the unshifted FFT of the DSB-SC signal.
2. Construct a USB mask that retains the positive-frequency interval fc to fc+W and its conjugate negative-frequency interval.
3. Construct an LSB mask that retains the positive-frequency interval fc-W to fc and its conjugate negative-frequency interval.
4. Multiply the DSB-SC spectrum by each mask and calculate the inverse FFT.
5. Plot the USB and LSB spectra.
6. Confirm that each SSB signal occupies W hertz.
7. Record the desired and undesired spectral amplitudes.

```
W = 35;
N = length(t);
F = (0:N-1)*(Fs/N);
F(F>=Fs/2) = F(F>=Fs/2)-Fs;
S = fft(sDSB);
maskUSB = ((F>=fc & F<=fc+W) | (F>=-(fc+W) & F<=-fc));
maskLSB = ((F>=fc-W & F<=fc) | (F>=-fc & F<=-(fc-W)));
sUSB_filter = real(ifft(S.*maskUSB));
sLSB_filter = real(ifft(S.*maskLSB));
```

Note: MATLAB does not provide a direct fftfreq command in all versions. Construct the unshifted frequency vector consistently with the FFT indexing used in your script, or use shifted spectra and masks with fftshift/ifftshift.

Part 3: Hilbert-Transform and Hartley-Modulator Method

1. Generate the analytic signal using analyticMessage=hilbert(m).
2. Obtain the Hilbert-transform signal mHat=imag(analyticMessage).
3. Plot m(t) and mHat(t) over a short time interval.
4. Compare their magnitude spectra and phase relationships.
5. Generate USB and LSB signals using the course-note equations.
6. Plot their time-domain waveforms and spectra.
7. Compare the Hilbert-method results with the frequency-discrimination results.
8. Calculate MSE and correlation between signals produced by the two methods. Account for any constant sign or delay only if it is clearly identified and justified.

```
mHat = imag(hilbert(m));
sUSB_hilbert = (Ac/2)*(m.*cos(2*pi*fc*t) ...
                    - mHat.*sin(2*pi*fc*t));
sLSB_hilbert = (Ac/2)*(m.*cos(2*pi*fc*t) ...
                    + mHat.*sin(2*pi*fc*t));
```

Part 4: Coherent Detection of USB and LSB

1. Generate the synchronized local oscillator 2cos(2π300t).
2. Multiply the USB signal by the local oscillator.
3. Apply a low-pass filter with a cutoff slightly above 35 Hz and well below the carrier frequency.
4. Divide the filtered result by Ac/2.
5. Repeat the same steps for the LSB signal.
6. Plot the original and recovered messages on the same axes.
7. Calculate the MSE, correlation coefficient, and recovered tone amplitudes.
8. Compare USB and LSB demodulation performance.

```
localOsc = 2*cos(2*pi*fc*t);
recUSB = lowpass(sUSB_hilbert.*localOsc,60,Fs)/(Ac/2);
recLSB = lowpass(sLSB_hilbert.*localOsc,60,Fs)/(Ac/2);
mseUSB = mean((m-recUSB).^2);
mseLSB = mean((m-recLSB).^2);
```

Part 5: Quadrature Phase Error and Sideband Suppression

1. Generate USB signals for quadrature carrier phase errors of 0°, 2°, 5°, 10°, and 20°.
2. Replace sin(2πfct) in the quadrature branch with sin(2πfct+ε).
3. Calculate the desired USB amplitudes at fc+15 Hz and fc+35 Hz.
4. Calculate the residual LSB amplitudes at fc-15 Hz and fc-35 Hz.
5. Calculate the sideband suppression ratio for each phase error.
6. Plot SSR versus phase error and complete Table 7.5.
7. Comment on the practical importance of accurate 90° phase shifting.

```
phaseError = [0 2 5 10 20];
for k = 1:length(phaseError)
    eps = deg2rad(phaseError(k));
    sErr{k} = (Ac/2)*(m.*cos(2*pi*fc*t) ...
              - mHat.*sin(2*pi*fc*t + eps));
end
```

Part 6: Final Comparison

1. Compare DSB-SC, USB, and LSB bandwidths.
2. Compare frequency-discrimination and Hilbert-transform SSB generation.
3. Compare message recovery from USB and LSB.

4. Explain the roles of the Hilbert transformer, quadrature carrier, adder/subtractor, product detector, and low-pass filter.
5. Complete all result tables and answer the post-laboratory questions.
6. Save the MATLAB script and figures using the naming convention specified by the laboratory supervisor.

6. Evaluation of Results

Tabulate and interpret the data obtained during the experiment.

Table 7.2. DSB-SC, USB, and LSB spectral components.

Signal	Theoretical positive-frequency components	FFT components	Measured bandwidth	Carrier present?
DSB-SC				
USB				
LSB				

Table 7.3. Comparison of SSB generation methods.

Signal	MSE between methods	Correlation coefficient	Desired sideband amplitude	Undesired sideband amplitude	Comment
USB					
LSB					

Table 7.4. Coherent SSB detection results.

Recovered signal	15 Hz amplitude	35 Hz amplitude	MSE	Correlation coefficient	LPF cutoff
From USB					
From LSB					

Table 7.5. Effect of quadrature phase error.

Phase error ε	Desired amplitude	Undesired amplitude	SSR (dB)	Observed spectral effect
0°				
2°				
5°				
10°				
20°				

Table 7.6. Bandwidth comparison.

Modulation type	Theoretical bandwidth	Measured bandwidth	Relative bandwidth
DSB-SC			
USB			
LSB			

Answer the following questions:

1. Why do the upper and lower sidebands contain equivalent message information, and why is the SSB bandwidth equal to W?
2. Which frequencies are retained in the USB and LSB spectra for the specified message?
3. Why is the carrier component absent from the ideal SSB spectrum?
4. Explain how a band-pass filter selects one sideband from a DSB-SC signal.
5. Why is sideband filtering difficult when the message spectrum extends close to zero frequency?
6. What amplitude and phase effects does the ideal Hilbert transformer apply?

7. Explain how the Hartley modulator and the plus/minus combination cancel different unwanted sidebands.
8. Compare the frequency-discrimination and phase-shift methods in terms of the required operations.
9. Why does coherent detection recover the same message from either USB or LSB?
10. Which high-frequency components are removed by the receiver low-pass filter?
11. What is sideband suppression ratio, and why does quadrature phase error reduce it?
12. How would amplitude mismatch between the in-phase and quadrature paths affect sideband suppression?
13. Compare SSB and DSB-SC in terms of bandwidth, transmitted spectral components, and implementation complexity.
14. Summarize the MATLAB evidence showing that the filter and Hilbert methods produce the same selected sideband.

Experiment 8: Vestigial-Sideband Modulation and Coherent Detection

1. Objective of the Experiment

The objective of this experiment is to investigate vestigial-sideband (VSB) modulation and coherent detection using MATLAB. Students will generate a double-sideband suppressed-carrier signal, apply a sideband-shaping filter that transmits one complete sideband together with a controlled vestige of the other sideband, and analyze the resulting signal in both the time and frequency domains.

The experiment emphasizes the bandwidth expression BWVSB = W + fv, the role of the vestigial transition region, and the additive-symmetry condition required for distortionless coherent recovery. The effects of vestigial bandwidth and filter asymmetry are evaluated by comparing the recovered waveform with the original message.

At the end of the experiment, students are expected to:

- Generate baseband, DSB-SC, and VSB signals in MATLAB.
- Identify the complete sideband and the residual vestigial sideband in the VSB spectrum.
- Calculate and verify the VSB transmission bandwidth.
- Design an idealized sideband-shaping filter with a controllable vestigial frequency.
- Explain the additive-symmetry condition H(fc+f)+H(fc-f)=constant.
- Recover the message by coherent detection and low-pass filtering.
- Quantify recovery quality using mean-square error and correlation.
- Compare DSB-SC, SSB, and VSB in terms of bandwidth and implementation requirements.

2. Theoretical Background

2.1. Principle of Vestigial-Sideband Modulation

A DSB-SC signal contains an upper and a lower sideband, each carrying equivalent information. SSB transmission removes one sideband completely, but the required sharp cutoff near the carrier may be difficult to realize when the message spectrum extends close to zero frequency. VSB modulation provides an intermediate solution by transmitting one complete sideband and only a small residual portion, called the vestige, of the opposite sideband.

$$sDSB\text{-}SC(t) = Ac\, m(t)\, cos(2\pi fct)$$

$$SDSB\text{-}SC(f) = (Ac/2)[M(f\text{-}fc) + M(f+fc)]$$

The DSB-SC spectrum is passed through a sideband-shaping filter H(f). The VSB spectrum is therefore

$$SVSB(f) = (Ac/2)[M(f\text{-}fc) + M(f+fc)]H(f)$$

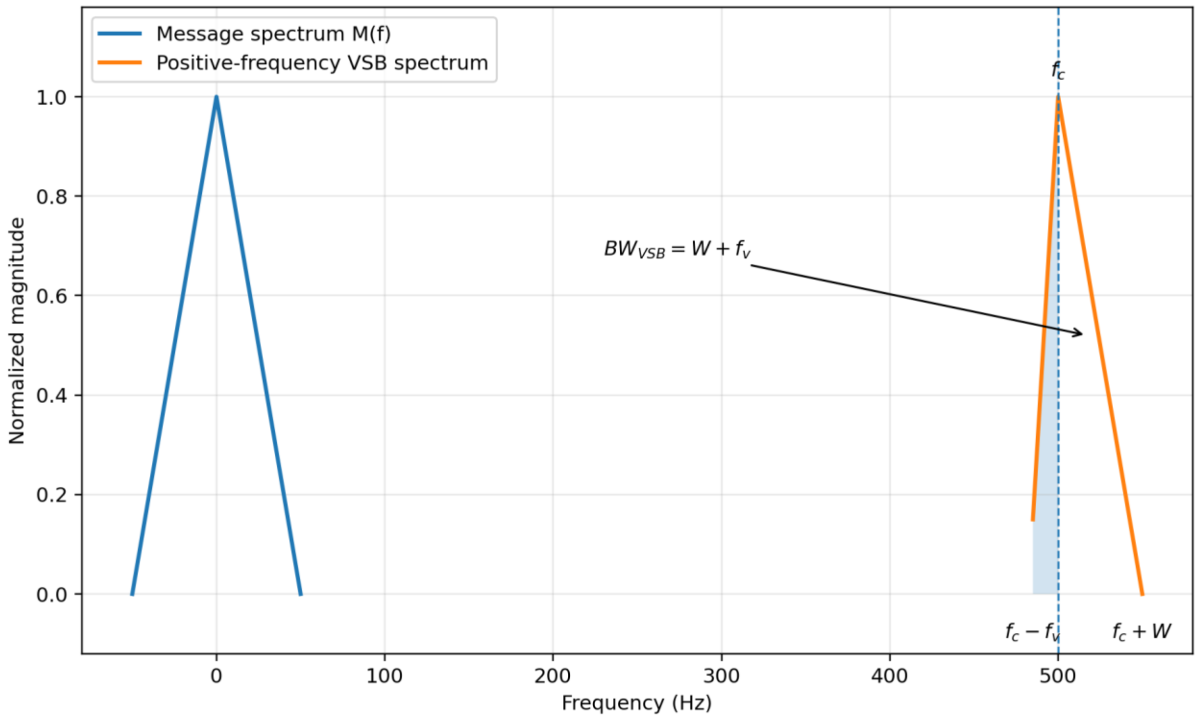

Fig. 8.1. Conceptual baseband and positive-frequency VSB spectra.

2.2. VSB Bandwidth

Let W denote the message bandwidth and fv denote the width of the retained vestigial portion. A VSB signal contains one full sideband of width W and an additional vestigial region of width fv. The transmission bandwidth is

$$BWVSB = W + fv$$

The limiting cases connect VSB to the neighboring modulation formats. When fv approaches zero, the spectrum approaches ideal SSB. When fv approaches W, both sidebands are nearly transmitted and the spectrum approaches DSB-SC.

2.3. VSB Transmitter and Receiver

The course-note structure generates DSB-SC with a product modulator and then applies a sideband-shaping filter. At the receiver, coherent multiplication by a synchronized carrier translates the desired spectrum back to baseband. A low-pass filter removes the components centered near twice the carrier frequency.

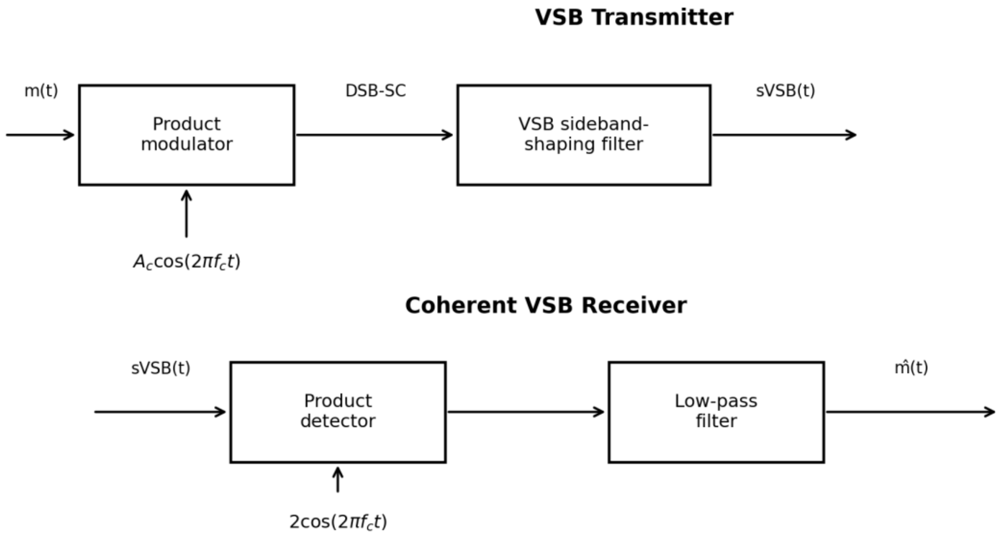

Fig. 8.2. VSB transmitter and coherent receiver block diagrams.

2.4. Coherent Detection and Additive Symmetry

Assume that the received VSB signal is multiplied by a synchronized local oscillator. After low-pass filtering, the baseband spectrum is proportional to

$$V0(f) = (Ac/2)M(f)[H(fc+f) + H(fc-f)]$$

For distortionless recovery, the term multiplying M(f) must remain constant over the entire message band. Thus, the VSB shaping filter must satisfy the additive-symmetry condition

$$H(fc+f) + H(fc-f) = constant, \quad |f| \leq W$$

A convenient normalized design uses a constant value of one. In the transition interval around the carrier, the two complementary responses then add to unity. If this condition is violated, different message frequencies experience different gains and the recovered waveform becomes distorted.

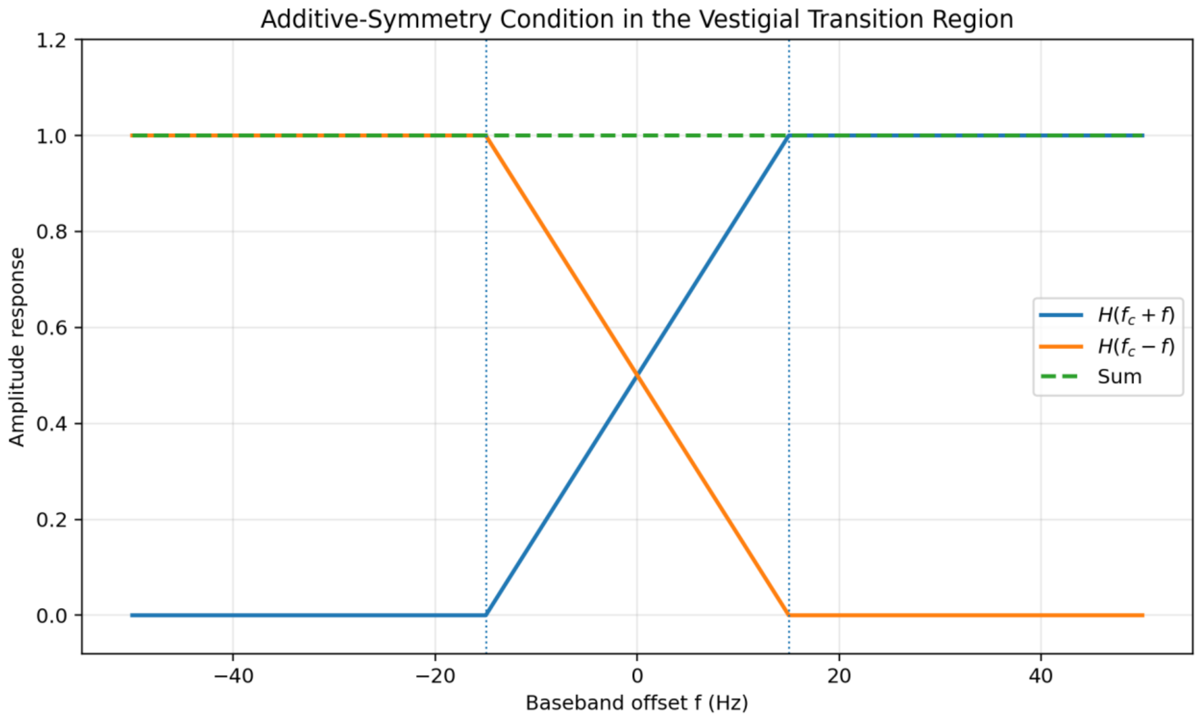

Fig. 8.3. Complementary VSB filter responses satisfying additive symmetry.

2.5. Time-Domain Interpretation

The course notes express the VSB signal as a combination of in-phase and quadrature components. The exact quadrature term depends on the selected shaping response. This representation shows that a general VSB waveform does not necessarily have an envelope directly proportional to the message. Coherent detection is therefore used in this experiment to provide controlled and theoretically verifiable recovery.

$$sVSB(t) = (Ac/2)m(t)cos(2\pi fct) - (Ac/2)mq(t)sin(2\pi fct)$$

Here mq(t) denotes the quadrature component produced by the vestigial sideband-shaping operation.

2.6. Comparison with DSB-SC and SSB

For a message bandwidth W, the ideal transmission bandwidths are

$$BWDSB\text{-}SC = 2W, \quad BWVSB = W + fv, \quad BWSSB = W$$

VSB requires less bandwidth than DSB-SC while permitting a more gradual and realizable transition near the carrier than ideal SSB. Its principal design requirement is the complementary filter response needed for low-distortion detection.

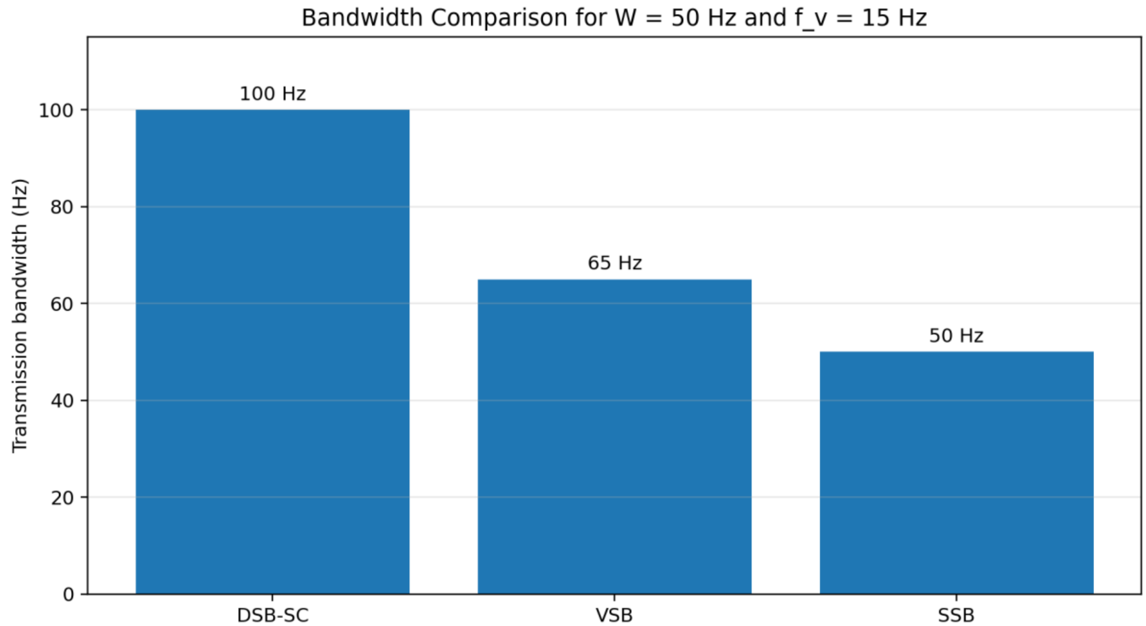

Fig. 8.4. DSB-SC, VSB, and SSB bandwidth comparison for the experiment parameters.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Signal Processing Toolbox or equivalent FFT-domain filtering commands
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, abs, max, mean, corrcoef
- fft, fftshift, ifft, ifftshift
- logical indexing and piecewise filter-mask construction
- plot, stem, subplot, grid, legend, xlim
- lowpass or an equivalent FFT-domain ideal low-pass filter

4. Preliminary (PreLab)

Use the following parameters throughout the preliminary calculations unless otherwise stated:

$$m(t) = \cos(2\pi 20t) + 0.6\cos(2\pi 50t)$$

$$Ac = 2, \quad fc = 500\ Hz, \quad Fs = 20\ kHz, \quad fv = 15\ Hz$$

Complete the following tasks before attending the laboratory:

1. Determine the message bandwidth W.

2. List all positive-frequency DSB-SC components.

3. Identify which positive-frequency components belong to the upper and lower sidebands.

4. Calculate the theoretical DSB-SC, VSB, and SSB bandwidths.

5. Determine the lower and upper positive-frequency edges of the VSB signal when the complete upper sideband is retained.

6. Sketch a normalized shaping response that is zero below fc-fv, rises through the vestigial transition region, and becomes one above fc+fv.

7. Show that a pair of complementary linear transition responses can satisfy H(fc+f)+H(fc-f)=1.

8. Derive the baseband term obtained after coherent multiplication and low-pass filtering.

9. Predict the effect of increasing fv from 15 Hz to 30 Hz on transmission bandwidth.

10. Explain why violating additive symmetry causes amplitude distortion in the recovered message.

Table 8.1. Preliminary spectral and bandwidth calculations.

Quantity	Expression	Calculated value
Message bandwidth	W	
Lower VSB edge	fc-fv	
Carrier frequency	fc	
Upper VSB edge	fc+W	
DSB-SC bandwidth	2W	
VSB bandwidth	W+fv	
SSB bandwidth	W	

5. Procedure

Part 1: Message and DSB-SC Generation

1. Open MATLAB and create a new script file.

2. Set the sampling frequency to 20 kHz and define a one-second observation interval.

3. Generate the two-tone message $m(t)=\cos(2\pi 20t)+0.6\cos(2\pi 50t)$.

4. Generate the carrier $A_c \cos(2\pi 500t)$ and the DSB-SC signal $s_{DSB}(t)=A_c\, m(t)\cos(2\pi 500t)$.

5. Plot the message and DSB-SC signals in the time domain.

6. Calculate centered and normalized FFTs.

7. Identify the DSB-SC components at fc±20 Hz and fc±50 Hz.

```
Fs = 20000;
t = 0:1/Fs:1-1/Fs;
Ac = 2; fc = 500; fv = 15;
m = cos(2*pi*20*t) + 0.6*cos(2*pi*50*t);
sDSB = Ac*m.*cos(2*pi*fc*t);
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
SDSB = fftshift(fft(sDSB))/N;
```

Part 2: Construction of the VSB Sideband-Shaping Filter

1. Set the message bandwidth to W=50 Hz and the vestigial frequency to fv=15 Hz.

2. Construct the unshifted frequency vector associated with fft.

3. Define the offset $\nu=|f|-fc$ around both positive and negative carrier frequencies.

4. Set the filter response to zero for $\nu \leq -fv$, use a linear transition for $-fv<\nu<fv$, and set the response to one for $fv \leq \nu \leq W$.

5. Keep the response equal to zero outside the translated message bands.

6. Plot H(f) around +fc and verify the transition edges.

7. Numerically verify that H(fc+f)+H(fc-f) remains approximately one over $|f| \leq W$.

```
F = (0:N-1)*(Fs/N);
F(F >= Fs/2) = F(F >= Fs/2)-Fs;
nu = abs(F)-fc;
H = zeros(size(F));
inband = (nu >= -W) & (nu <= W);
H((nu <= -fv) & inband) = 0;
idx = (nu > -fv) & (nu < fv) & inband;
H(idx) = (nu(idx)+fv)/(2*fv);
H((nu >= fv) & inband) = 1;
```

Part 3: VSB Signal Generation and Spectral Analysis

1. Calculate the unshifted FFT of the DSB-SC signal.

2. Multiply the DSB-SC spectrum by the VSB shaping filter.

3. Obtain the real-valued VSB signal using the inverse FFT.

4. Plot the VSB signal over a short time interval.

5. Plot the positive-frequency spectrum from 400 Hz to 570 Hz.

6. Identify the complete upper sideband and the vestigial portion of the lower sideband.

7. Measure the occupied bandwidth and compare it with W+fv.

```
S = fft(sDSB);
SVSB = S.*H;
sVSB = real(ifft(SVSB));
SVSBc = fftshift(SVSB)/N;
subplot(2,1,1); plot(t,sVSB); xlim([0 0.08]); grid on;
subplot(2,1,2); plot(f,abs(SVSBc)); xlim([400 570]); grid on;
```

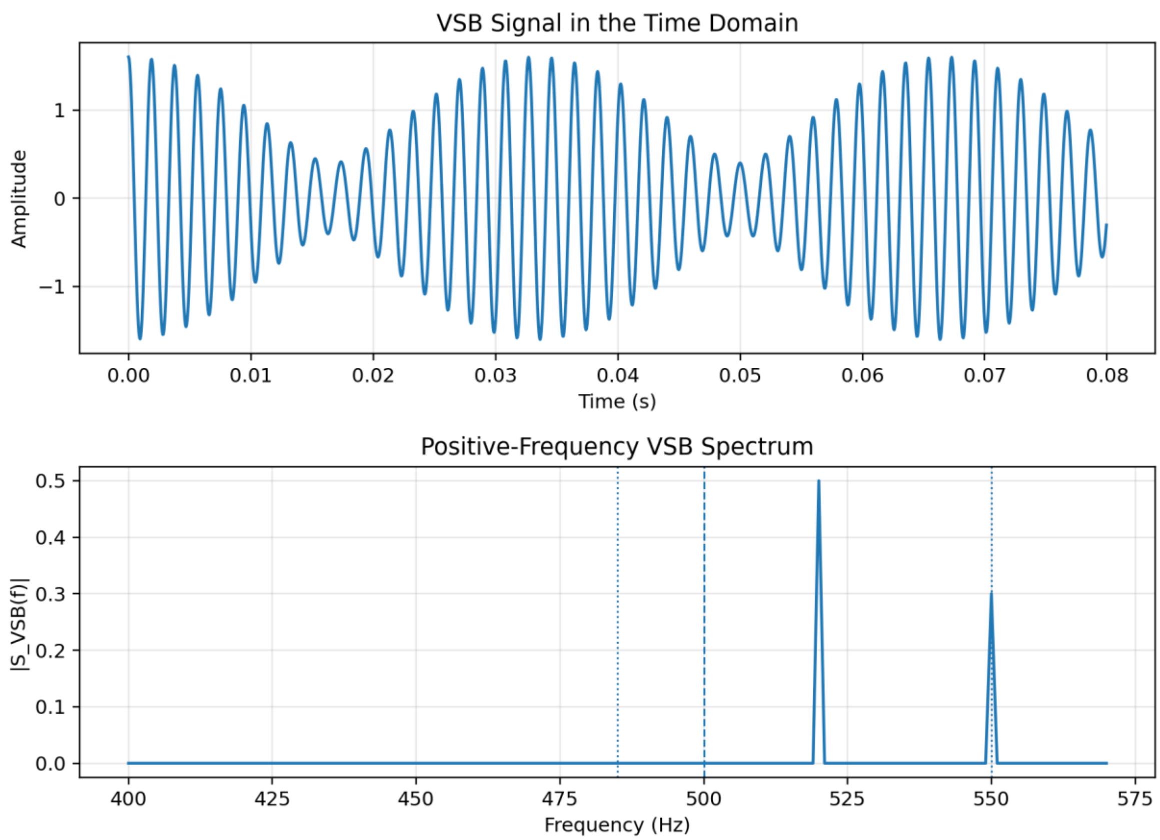

Fig. 8.5. Representative VSB time-domain waveform and positive-frequency spectrum.

Part 4: Effect of the Vestigial Frequency

1. Repeat the shaping-filter and VSB-generation steps for fv=5, 15, 30, and 50 Hz.

2. Plot all shaping responses on the same axes.

3. Determine the measured transmission bandwidth for each case.

4. Observe the progression from an SSB-like spectrum toward a DSB-SC-like spectrum.

5. Record the results in Table 8.3.

Part 5: Coherent Detection of the VSB Signal

1. Generate the synchronized local oscillator 2cos(2π500t).

2. Multiply the VSB signal by the local oscillator.

3. Apply a low-pass filter with a cutoff frequency of 70 Hz.

4. Scale the filtered result by 2/Ac to obtain the recovered message.

5. Plot the original and recovered messages on the same axes.

6. Compare the 20 Hz and 50 Hz spectral amplitudes.

7. Calculate mean-square error and correlation coefficient.

8. Complete Table 8.4.

```
mixed = 2*sVSB.*cos(2*pi*fc*t);
recovered = lowpass(mixed,70,Fs)*(2/Ac);
```

```
mseValue = mean((m-recovered).^2);
C = corrcoef(m,recovered);
correlation = C(1,2);
```

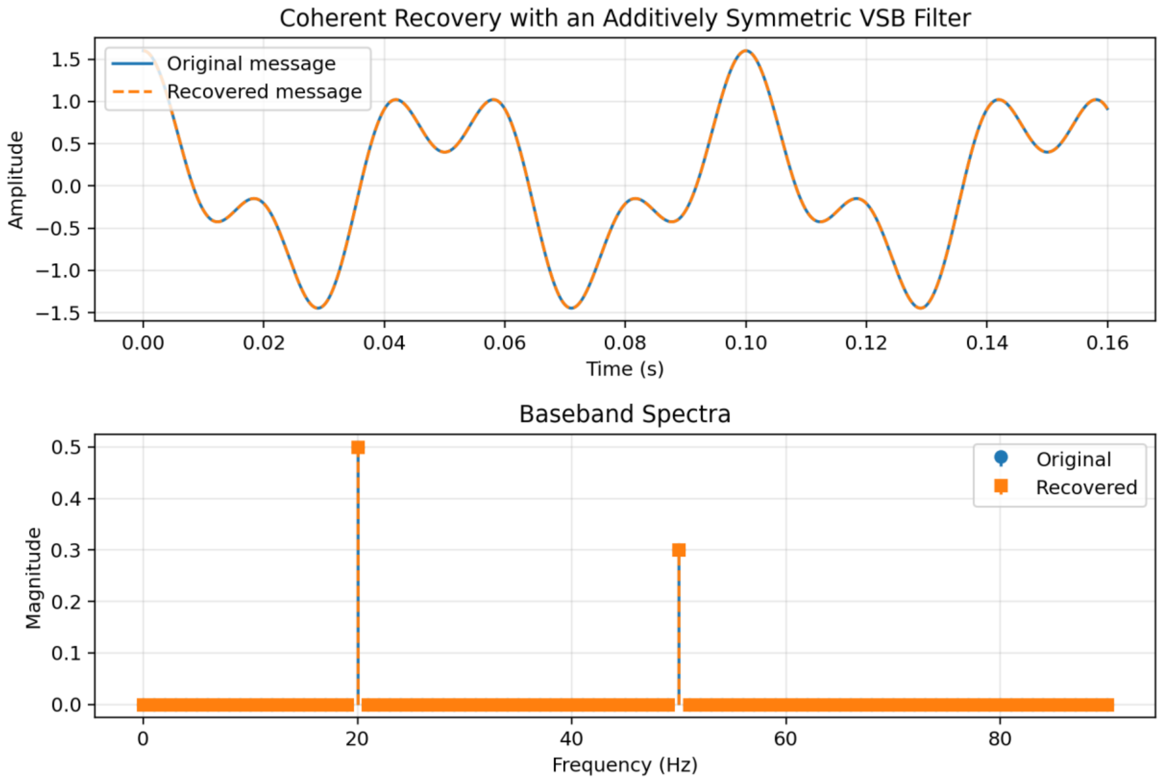

Fig. 8.6. Coherent recovery of the message with an additively symmetric VSB filter.

Part 6: Violation of the Additive-Symmetry Condition

1. Modify the transition response by shifting its center or changing only one side of the transition.

2. Plot H(fc+f)+H(fc-f) for the modified filter.

3. Generate the corresponding VSB signal and recover the message using the same coherent receiver.

4. Compare the distorted recovery with the symmetric-filter recovery.

5. Calculate the MSE and correlation for both cases.

6. Explain which message tone is affected more strongly and relate the result to the nonconstant summed response.

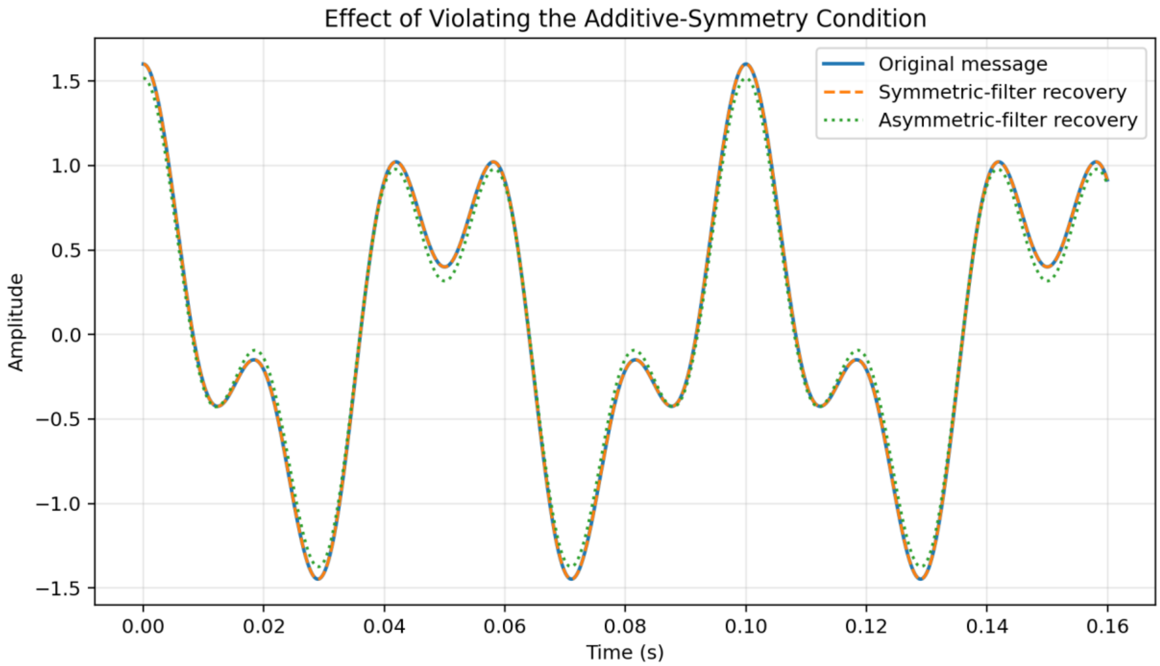

Fig. 8.7. Example distortion produced by a VSB filter that violates additive symmetry.

Part 7: Final Comparison

1. Compare DSB-SC, VSB, and SSB bandwidths for the same message.
2. Compare the spectral components transmitted by each method.
3. Explain why VSB relaxes the sharp-filter requirement of ideal SSB.
4. Explain why coherent recovery depends on the complementary transition responses.
5. Complete all result tables and answer the post-laboratory questions.
6. Save the MATLAB script and figures using the naming convention specified by the laboratory supervisor.

6. Evaluation of Results

Tabulate and interpret the data obtained during the experiment.

Table 8.2. DSB-SC and VSB spectral components.

Spectral component	Theoretical frequency	Measured frequency	Measured magnitude
Lower sideband, tone 1	fc-20		
Lower sideband, tone 2	fc-50		
Upper sideband, tone 1	fc+20		
Upper sideband, tone 2	fc+50		

Table 8.3. Effect of vestigial frequency on transmission bandwidth.

fv (Hz)	Theoretical BW = W+fv (Hz)	Measured lower edge (Hz)	Measured upper edge (Hz)	Measured BW (Hz)
5				
15				
30				
50				

Table 8.4. Coherent VSB detection results.

Quantity	Original message	Recovered message	Percentage error / result
20 Hz tone magnitude			
50 Hz tone magnitude			
RMS value			
Mean-square error	-		
Correlation coefficient	-		

Table 8.5. Effect of the VSB additive-symmetry condition.

Filter case	Max/min of H(fc+f)+H(fc-f)	MSE	Correlation	Observed distortion
Additively symmetric				
Asymmetric transition				

Table 8.6. Bandwidth comparison.

Modulation type	Theoretical bandwidth	Measured bandwidth	Relative bandwidth
DSB-SC	2W		
VSB	W+fv		
SSB	W		

Answer the following questions:

1. Why is VSB considered an intermediate modulation method between DSB-SC and SSB?
2. Which complete sideband and which vestigial sideband are retained by the filter used in this experiment?
3. Derive the VSB bandwidth expression BWVSB=W+fv.
4. What are the limiting spectral cases when fv approaches zero and when fv approaches W?
5. Why is ideal SSB filtering difficult for message spectra extending close to zero frequency?
6. Explain the functions of the product modulator and sideband-shaping filter in the VSB transmitter.

7. Which high-frequency components are generated by coherent multiplication, and why are they removed by the low-pass filter?

8. State and interpret the additive-symmetry condition for distortionless VSB detection.

9. Why must H(fc+f)+H(fc-f) remain constant over the message band?

10. How does increasing fv affect bandwidth and filter-transition requirements?

11. Compare the recovered 20 Hz and 50 Hz tone amplitudes for the symmetric and asymmetric filters.

12. Why is the envelope of a general suppressed-carrier VSB signal not necessarily proportional to the message?

13. Compare DSB-SC, VSB, and SSB in terms of bandwidth, filter complexity, and transmitted spectral content.

14. Summarize the MATLAB evidence demonstrating that additive symmetry provides low-distortion coherent recovery.

Experiment 9: Frequency and Phase Modulation

1. Objective of the Experiment

The objective of this experiment is to investigate angle modulation by generating and analyzing frequency-modulated (FM) and phase-modulated (PM) signals in MATLAB. Students will relate the message signal to the instantaneous phase and instantaneous frequency of a constant-amplitude carrier, observe the characteristic time-domain waveforms, and evaluate the effect of modulation sensitivity and modulation index on the signal spectrum.

The experiment also examines single-tone FM, frequency deviation, the FM modulation index, Carson's bandwidth rule, and the distinction between narrowband and wideband FM. FM and PM are compared to show that FM may be produced by integrating the message before phase modulation, whereas PM may be produced by differentiating the message before frequency modulation.

At the end of the experiment, students are expected to:

- Define instantaneous phase and instantaneous frequency for a sinusoidal carrier.
- Generate single-tone FM and PM signals using MATLAB.
- Calculate frequency deviation and the FM modulation index.
- Explain how the sign and amplitude of the message alter the instantaneous carrier frequency.
- Compare FM and PM in the time and frequency domains.
- Observe spectral broadening as the modulation index increases.
- Estimate FM bandwidth using Carson's rule.
- Distinguish narrowband FM from wideband FM.
- Verify the integration/differentiation relationship between FM and PM.

2. Theoretical Background

2.1. Angle-Modulated Carrier

A sinusoidal carrier may be written in terms of its amplitude and total phase as

$$x(t) = Ac\ cos[theta(t)] = Ac\ cos[2\pi fct + phi(t)]$$

In angle modulation, the carrier amplitude Ac remains constant while the instantaneous angle is varied by the message. The instantaneous angular frequency is the derivative of total phase, and total phase is the integral of instantaneous angular frequency.

$$omega\_i(t) = dtheta(t)/dt$$

$$theta(t) = \text{integral from -infinity to t of } omega\_i(tau)\ d\ tau$$

Frequency modulation changes the instantaneous frequency, whereas phase modulation changes the instantaneous phase. The difference is therefore determined by how the message enters the carrier angle.

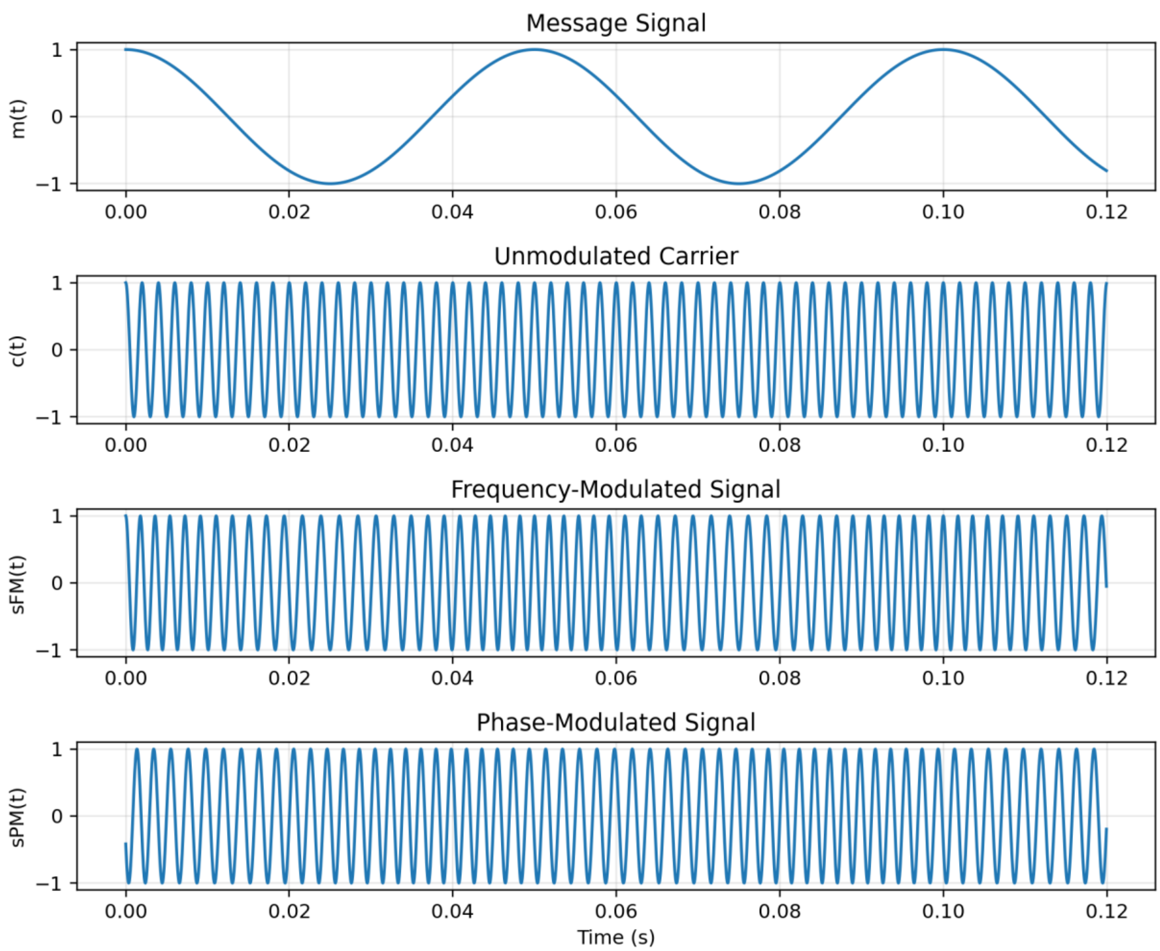

Fig. 9.1. Message, unmodulated carrier, FM signal, and PM signal in the time domain.

2.2. Frequency Modulation

For frequency modulation, the instantaneous frequency is varied linearly with the message signal. Using frequency sensitivity kf, the instantaneous frequency is

$$fi(t) = fc + kf\ m(t)$$

The corresponding FM phase is obtained by integrating the frequency deviation. The general FM signal is

$$sFM(t) = Ac\ cos[2\pi fct + 2\pi kf\ \text{integral from -infinity to t of}\ m(tau)\ d\ tau]$$

For a single-tone message

$$m(t) = Am\ cos(2\pi fmt)$$

the FM waveform becomes

$$sFM(t) = Ac\ cos[2\pi fct + beta\ sin(2\pi fmt)]$$

The peak frequency deviation and FM modulation index are

$$Delta\ f = kf\ Am$$

$$beta = Delta\ f\ /\ fm$$

When the message is positive, the instantaneous frequency increases above fc; when the message is negative, it decreases below fc. The carrier amplitude remains unchanged.

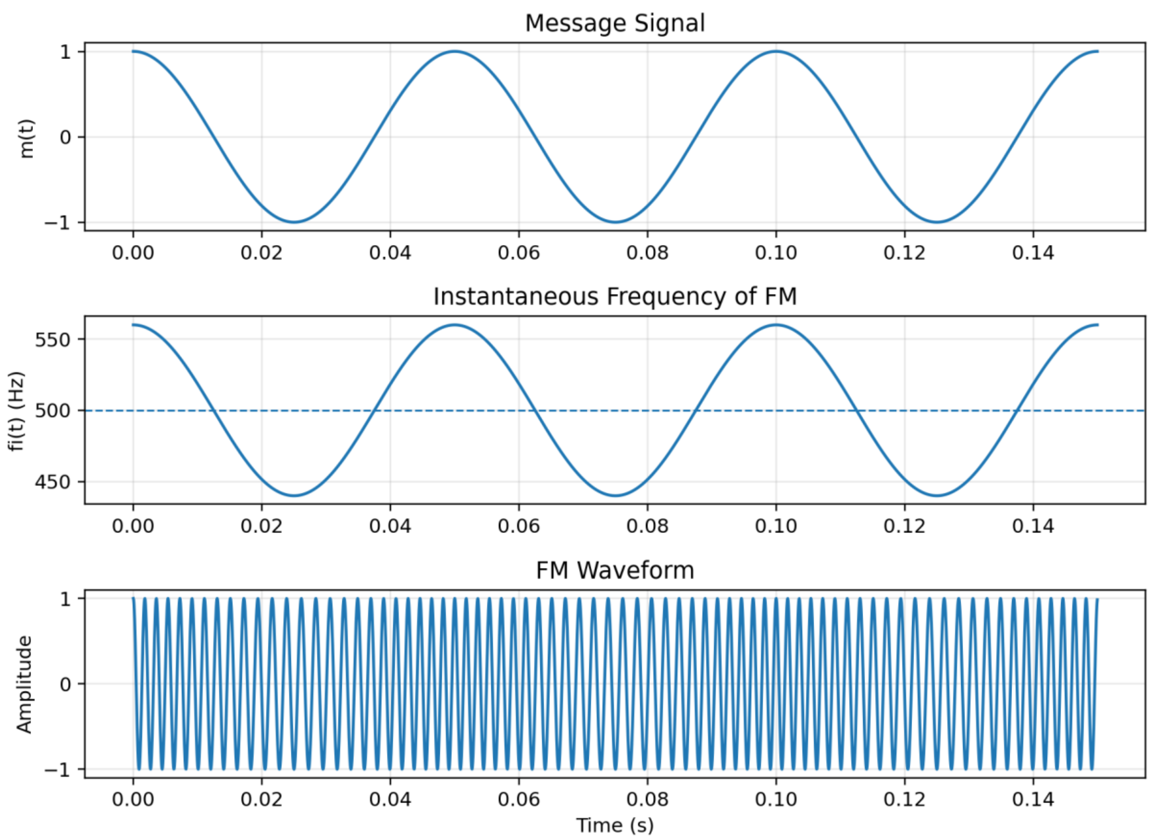

Fig. 9.2. Relationship between the message, instantaneous frequency, and FM waveform.

2.3. Phase Modulation

For phase modulation, the instantaneous phase deviation is proportional to the message signal. Using phase sensitivity kp, the PM signal is

$$sPM(t) = Ac\ cos[2\pi fct + kp\ m(t)]$$

The peak phase deviation is kpAm radians. Although PM directly varies phase, differentiating the PM phase shows that its instantaneous frequency also changes:

$$fi,PM(t) = fc + (kp/2\pi)\ dm(t)/dt$$

For m(t)=Am cos(2πfmt), this relationship becomes

$$fi,PM(t) = fc - kp\ Am\ fm\ sin(2\pi fmt)$$

Thus, for a fixed message amplitude and phase sensitivity, PM frequency deviation increases with message frequency. This differs from FM, for which Delta f=kfAm is independent of the single-tone message frequency.

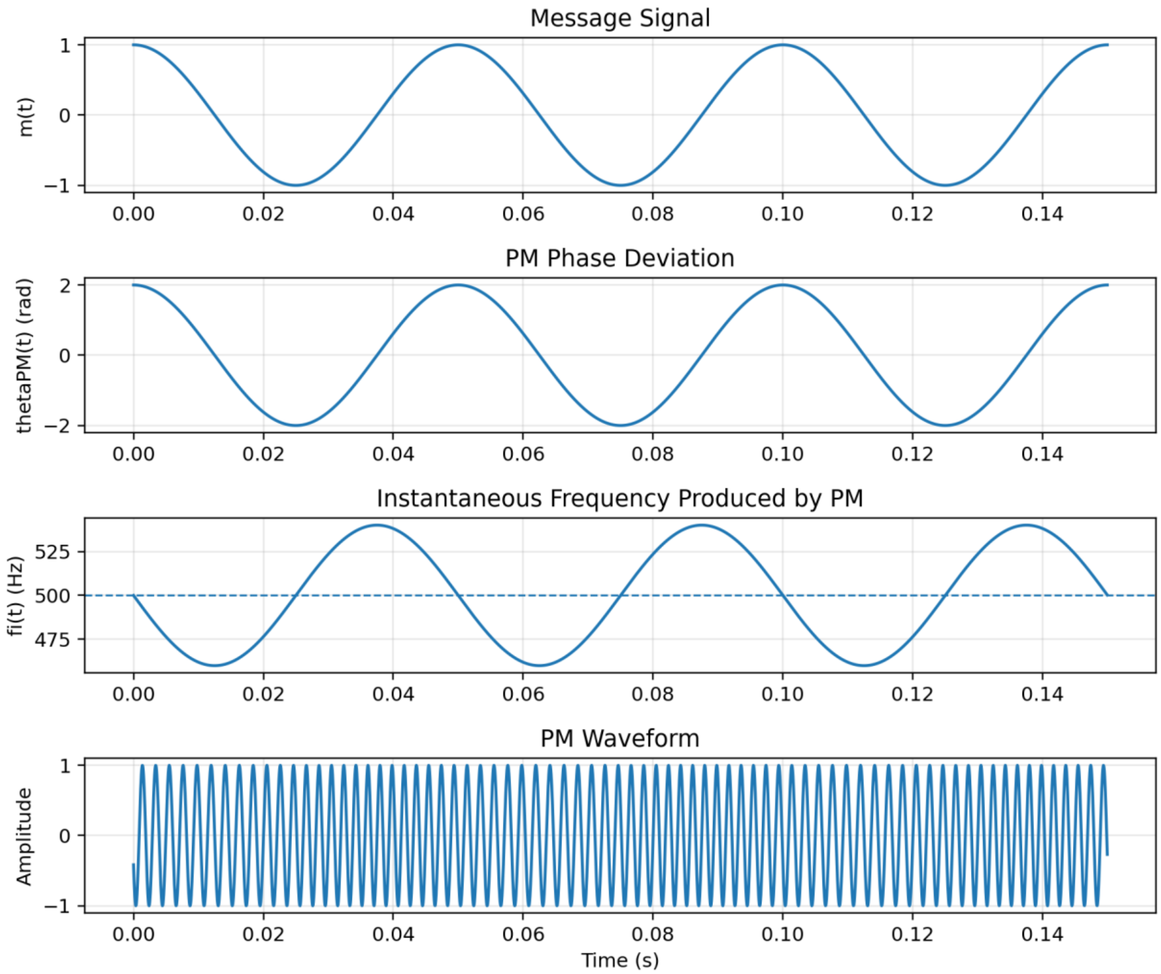

Fig. 9.3. Message, PM phase deviation, instantaneous frequency, and PM waveform.

2.4. Relationship Between FM and PM

Because phase is the integral of instantaneous angular frequency, FM can be generated by integrating the message and applying the result to a phase modulator. Conversely, PM can be generated by differentiating the message before applying it to a frequency modulator. These relationships are illustrated in the course notes and will be verified numerically in the procedure.

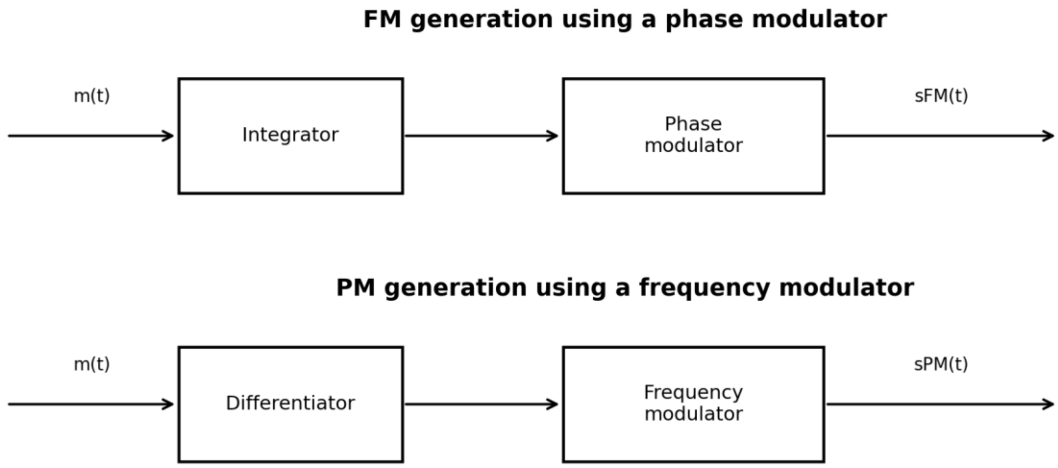

Fig. 9.4. Generation relationships between FM and PM.

2.5. FM Spectrum and Modulation Index

A single-tone FM signal generally contains a carrier component and many sideband pairs located at

$$f = fc \text{ plus or minus } n\ fm, \quad n = 0, 1, 2, \ldots$$

The number of visible sidebands increases as beta increases. For beta much smaller than one, the signal is referred to as narrowband FM and most of the energy is concentrated in the carrier and the first sideband pair. For beta much greater than one, many sideband pairs become important and the signal is referred to as wideband FM.

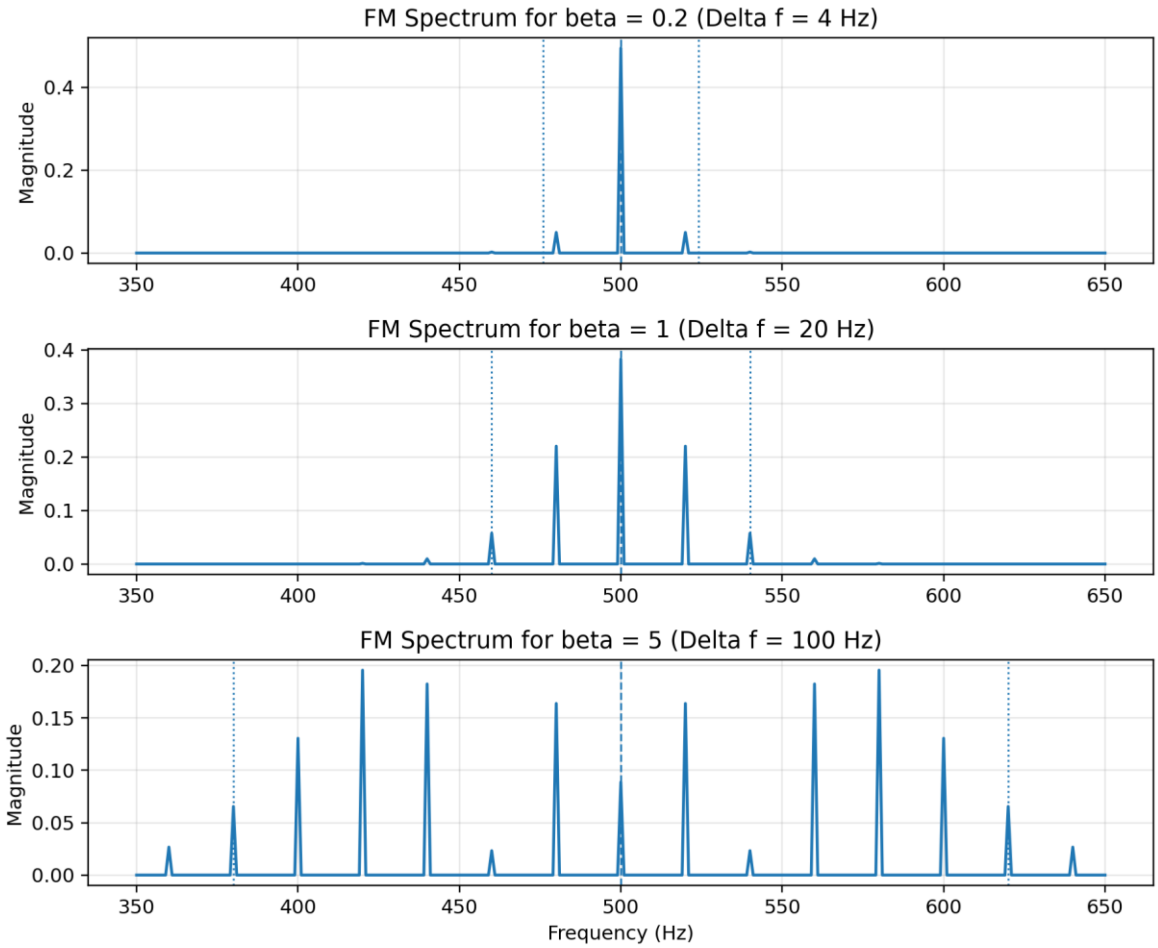

Fig. 9.5. Representative FM spectra for increasing modulation index.

2.6. Carson's Bandwidth Rule

The theoretical FM spectrum contains infinitely many sidebands. Carson's rule provides a practical estimate of the bandwidth containing the dominant components:

BWFM approximately 2(Delta f + fm)

Using beta=Delta f/fm, the same expression may be written as

BWFM approximately 2 Delta f (1 + 1/beta) = 2 fm (beta + 1)

For beta much smaller than one, the bandwidth approaches approximately 2fm. For beta much greater than one, the bandwidth is dominated by approximately 2Delta f. These are practical approximations associated with narrowband and wideband FM, respectively.

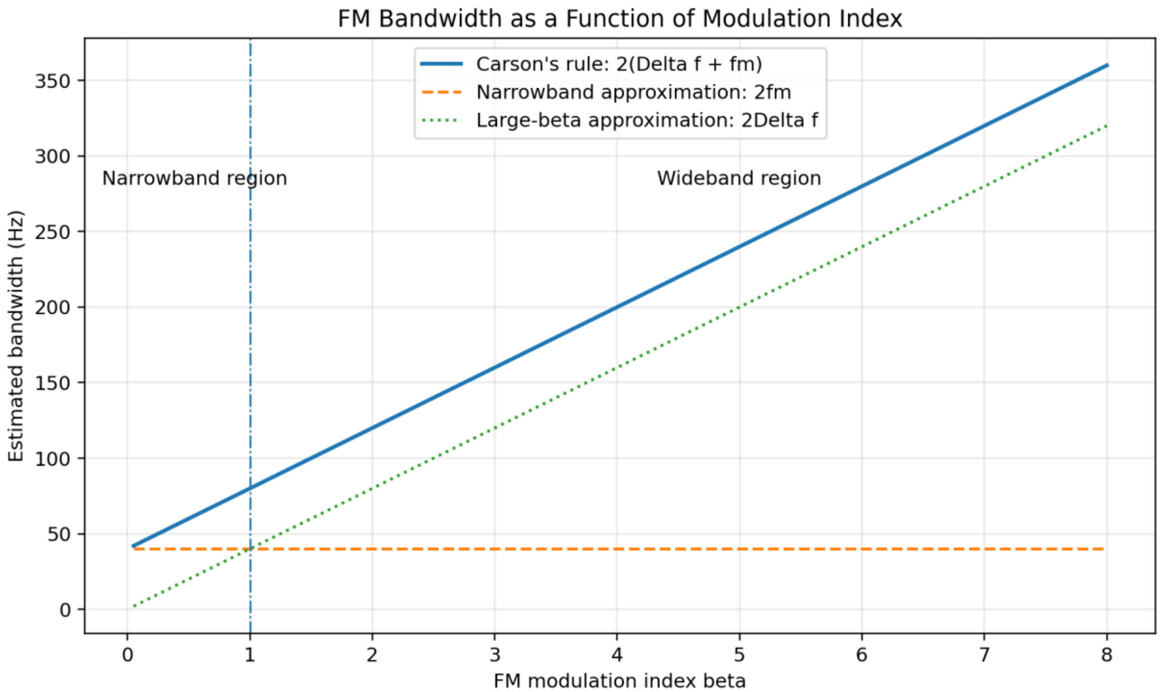

Fig. 9.6. Carson bandwidth estimate and narrowband/wideband limiting trends.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Script Editor
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, sin, pi, cumsum, diff, gradient
- fft, fftshift, abs, length
- unwrap and angle when phase estimation is required
- plot, stem, subplot, grid, legend, xlim
- mean, max, min, and logical indexing

4. Preliminary (PreLab)

Use the following reference parameters unless otherwise stated:

$$Ac = 1, \quad Am = 1, \quad fc = 500\ Hz, \quad fm = 20\ Hz, \quad Fs = 20\ kHz$$

Complete the following tasks before attending the laboratory:

1. Write the mathematical expressions for the message, carrier, single-tone FM, and single-tone PM signals.

2. For kf=60 Hz/unit, calculate the peak frequency deviation Delta f and the FM modulation index beta.

3. Use Carson's rule to calculate the estimated bandwidth for beta=0.2, beta=1, and beta=5.

4. Calculate the corresponding lower and upper Carson frequency limits around fc for each beta.

5. For kp=2 rad/unit, calculate the peak PM phase deviation.

6. For the PM signal, calculate the peak instantaneous frequency deviation when fm=20 Hz.

7. Repeat the PM frequency-deviation calculation for fm=40 Hz and explain the change.

8. Sketch the instantaneous FM frequency for one period of the cosine message.

9. State the expected spectral line spacing of single-tone FM.

10. Explain the conditions beta<<1 and beta>>1 in terms of narrowband and wideband FM.

11. Explain how an integrator followed by a phase modulator produces FM.

12. Explain how a differentiator followed by a frequency modulator produces PM.

Table 9.1. Preliminary single-tone FM calculations.

Case	beta	Delta f (Hz)	Carson BW (Hz)	Lower limit (Hz)	Upper limit (Hz)
Narrowband case	0.2				
Intermediate case	1				
Wideband case	5				

Table 9.2. Preliminary PM calculations.

PM parameter	Expression	Calculated value
Peak phase deviation	kpAm	
Peak frequency deviation at 20 Hz	kpAmfm	
Peak frequency deviation at 40 Hz	kpAmfm	

5. Procedure

Part 1: Generation of the Message and Carrier Signals

1. Open MATLAB and create a new script file.

2. Set Fs=20 kHz and define a one-second observation interval.

3. Generate m(t)=cos(2π20t) and c(t)=cos(2π500t).

4. Plot the message and carrier signals in separate subplots.

5. Confirm that the carrier amplitude remains constant.

```
Fs = 20000;
t = 0:1/Fs:1-1/Fs;
Ac = 1; Am = 1; fc = 500; fm = 20;
m = Am*cos(2*pi*fm*t);
c = Ac*cos(2*pi*fc*t);
```

Part 2: Single-Tone FM Generation

1. Select beta=3 and calculate Delta f=beta fm.

2. Generate the FM signal sFM(t)=Ac cos[2πfct+beta sin(2πfmt)].

3. Calculate the theoretical instantaneous frequency fi(t)=fc+Delta f cos(2πfmt).

4. Plot m(t), fi(t), and sFM(t) using subplot.

5. Identify the times at which the instantaneous frequency is maximum, equal to fc, and minimum.

6. Verify that the FM amplitude remains constant.

```
beta = 3;
DeltaF = beta*fm;
sFM = Ac*cos(2*pi*fc*t + beta*sin(2*pi*fm*t));
fiFM = fc + DeltaF*cos(2*pi*fm*t);
subplot(3,1,1); plot(t,m); xlim([0 0.15]); grid on;
subplot(3,1,2); plot(t,fiFM); xlim([0 0.15]); grid on;
subplot(3,1,3); plot(t,sFM); xlim([0 0.15]); grid on;
```

Part 3: Effect of the FM Modulation Index

1. Generate FM signals for beta=0.2, beta=1, and beta=5 while keeping fm=20 Hz and fc=500 Hz.

2. Calculate a centered, normalized FFT for each signal.

3. Plot the three magnitude spectra from 350 Hz to 650 Hz.

4. Identify the carrier and sidebands separated by integer multiples of fm.

5. Count the clearly visible sideband pairs for each beta.

6. Calculate Carson bandwidth and compare it with the significant spectral region observed in MATLAB.

7. Complete Tables 9.3 and 9.4.

```
betaList = [0.2 1 5];
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
for k = 1:length(betaList)
    beta = betaList(k);
    s = Ac*cos(2*pi*fc*t + beta*sin(2*pi*fm*t));
    S = fftshift(fft(s))/N;
    subplot(3,1,k);
    plot(f,abs(S)); xlim([350 650]); grid on;
end
```

Part 4: Verification of Carson's Rule

1. For each beta, calculate Delta f=beta fm.

2. Calculate BW=2(Delta f+fm).

3. Mark the theoretical limits fc-BW/2 and fc+BW/2 on the FFT plots.

4. Define a consistent magnitude threshold and estimate the occupied bandwidth from the numerical spectrum.

5. Compare theoretical and measured bandwidths and discuss why they need not be identical.

6. Classify each case as narrowband, transitional, or wideband FM.

Part 5: Phase Modulation

1. Set kp=2 rad/unit and generate sPM(t)=Ac cos[2πfct+kp m(t)].

2. Calculate thetaPM(t)=kp m(t).

3. Calculate fi,PM(t)=fc+(kp/2π)dm(t)/dt using the analytical derivative or MATLAB gradient.

4. Plot m(t), thetaPM(t), fi,PM(t), and sPM(t).

5. Identify the message values at which PM phase deviation is maximum and minimum.

6. Identify the times at which the PM instantaneous frequency has its maximum deviation.

7. Repeat the calculation for fm=40 Hz while keeping Am and kp unchanged.

8. Compare the PM frequency deviations at 20 Hz and 40 Hz.

```
kp = 2;
thetaPM = kp*m;
sPM = Ac*cos(2*pi*fc*t + thetaPM);
dm_dt = gradient(m,1/Fs);
fiPM = fc + (kp/(2*pi))*dm_dt;
```

Part 6: Numerical Verification of the FM-PM Relationship

1. Numerically integrate the message signal using cumsum(m)/Fs.

2. Apply the integrated message to a phase-modulation expression with a suitable scaling constant.

3. Generate a direct FM signal using the equivalent frequency-sensitivity parameter.

4. Plot the two signals and calculate their mean-square difference.

5. Numerically differentiate the message using gradient.

6. Apply the differentiated message to a frequency-modulation expression.

7. Compare the result with direct PM and comment on numerical edge effects.

```
mInt = cumsum(m)/Fs;
kPhase = 2*pi*60;
sFM_from_PM = Ac*cos(2*pi*fc*t + kPhase*mInt);
sFM_direct = Ac*cos(2*pi*fc*t + 2*pi*60*cumsum(m)/Fs);
mseFM = mean((sFM_from_PM-sFM_direct).^2);
```

Part 7: Final Comparison of FM and PM

1. Plot representative FM and PM waveforms on the same time interval.

2. Compare which message quantity controls FM instantaneous frequency and PM instantaneous phase.

3. Compare the dependence of their frequency deviations on message amplitude and message frequency.

4. Confirm that both modulation types preserve constant carrier amplitude.

5. Save the MATLAB script, FFT figures, tables, and written interpretations using the naming convention specified by the laboratory supervisor.

6. Evaluation of Results

Tabulate and interpret the data obtained during the experiment.

Table 9.3. FM modulation-index and bandwidth results.

beta	Delta f theoretical (Hz)	Carson BW (Hz)	Measured BW (Hz)	Number of visible sideband pairs
0.2				
1				
5				

Table 9.4. FM spectral measurements.

beta	Carrier frequency measured (Hz)	Sideband spacing measured (Hz)	Lower Carson limit (Hz)	Upper Carson limit (Hz)
0.2				
1				
5				

Table 9.5. FM time-domain verification.

Quantity	Theoretical result	MATLAB result	Percentage error / comment
Maximum FM instantaneous frequency			
Minimum FM instantaneous frequency			
FM peak frequency deviation			
FM signal peak amplitude			

Table 9.6. PM phase and instantaneous-frequency results.

PM case	Peak phase deviation (rad)	Peak frequency deviation (Hz)	Measured deviation (Hz)	Comment
fm=20 Hz				
fm=40 Hz				

Table 9.7. Numerical verification of the FM-PM relationships.

Relationship test	MSE	Maximum absolute difference	Interpretation
FM generated through integration and phase modulation			
PM generated through differentiation and frequency modulation			

Answer the following questions:

1. What carrier parameter remains constant in ideal FM and PM?
2. Define instantaneous phase and instantaneous frequency and state their mathematical relationship.
3. How does the sign of m(t) affect the FM instantaneous frequency?
4. Derive the single-tone FM expression from fi(t)=fc+kfm(t).
5. Define frequency deviation and the FM modulation index.
6. Why are FM spectral components separated by fm?
7. How does increasing beta change the number of visible sidebands?
8. State Carson's bandwidth rule and explain its practical purpose.

9. Why does narrowband FM have an approximate bandwidth near 2fm?

10. Why does wideband FM bandwidth become dominated by approximately 2Delta f?

11. For PM, why does the instantaneous frequency depend on dm(t)/dt?

12. Why does doubling the message frequency increase PM frequency deviation when Am and kp remain constant?

13. Compare FM and PM with respect to message amplitude and message frequency.

14. Explain how an integrator and phase modulator can be used to produce FM.

15. Explain how a differentiator and frequency modulator can be used to produce PM.

16. Discuss possible differences between Carson bandwidth and a threshold-based FFT bandwidth.

17. Summarize the MATLAB evidence distinguishing narrowband and wideband FM.

Experiment 10: Comparative Analysis of Analog Modulation Techniques

1. Objective of the Experiment

The objective of this final laboratory experiment is to compare the principal analog modulation techniques studied throughout the Introduction to Communication Systems course by implementing them under a common set of signal parameters in MATLAB. Conventional amplitude modulation (AM), double-sideband suppressed-carrier modulation (DSB-SC), single-sideband modulation (SSB), vestigial-sideband modulation (VSB), frequency modulation (FM), and phase modulation (PM) are generated and examined in the time and frequency domains.

The experiment consolidates the Fourier-transform, frequency-translation, sideband, coherent-detection, envelope-detection, and angle-modulation concepts developed in the previous experiments. The comparison focuses on carrier presence, sideband structure, theoretical bandwidth, mean transmitted power, recovery method, and the characteristic parameter varied by each modulation technique.

At the end of the experiment, students are expected to:

- Generate AM, DSB-SC, SSB, VSB, FM, and PM signals using common message and carrier parameters.
- Identify the carrier and sideband components of each linear modulation technique.
- Compare the time-domain envelopes of amplitude-modulated and angle-modulated signals.
- Calculate theoretical transmission bandwidths and verify them from FFT results.
- Compare carrier transmission and mean signal power among the modulation techniques.
- Recover the message from AM using envelope detection and from DSB-SC, SSB, and VSB using coherent detection.
- Explain why coherent carrier synchronization is required for suppressed-carrier signals.
- Distinguish linear modulation from angle modulation in terms of the carrier parameter that changes.
- Summarize the advantages and limitations of each method using numerical evidence.

2. Theoretical Background

2.1. Classification of Analog Modulation Techniques

A communication system translates a baseband message to a higher carrier frequency so that the signal can be transmitted through a bandpass channel. In linear amplitude modulation, the message changes the amplitude or sideband structure of the carrier. In angle modulation, the carrier amplitude remains constant while the instantaneous phase or instantaneous frequency is varied.

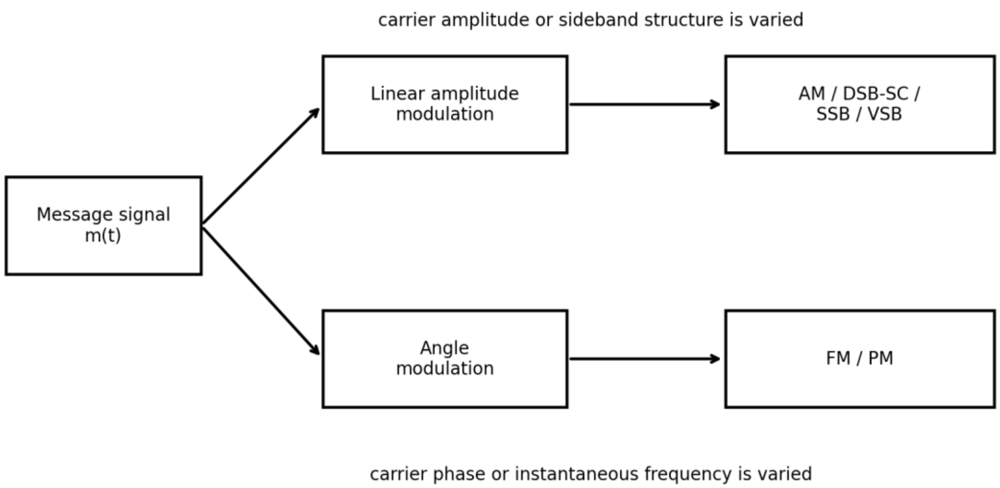

Fig. 10.1. Classification of the analog modulation techniques compared in the experiment.

2.2. Conventional Amplitude Modulation

For a normalized message m(t) with $|m(t)|\leq 1$, a conventional AM signal is

$$s_{AM}(t) = A_c[1 + \mu m(t)]\cos(2\pi f_c t)$$

where μ is the modulation index. The spectrum contains a carrier at f_c and two sidebands translated to $f_c \pm f_m$ for a single-tone message. To avoid overmodulation, the condition $0\leq\mu\leq 1$ must be satisfied. For a message bandwidth W, the transmission bandwidth is

$$BW_{AM} = 2W$$

The presence of the carrier allows envelope detection, but the carrier consumes power without carrying new message information.

2.3. DSB-SC Modulation

In DSB-SC modulation, the message directly multiplies the carrier:

$$s_{DSB\text{-}SC}(t) = A_c\, m(t)\cos(2\pi f_c t)$$

The carrier component is suppressed and only the upper and lower sidebands are transmitted. The bandwidth remains 2W, but coherent detection is required because the envelope is not generally equal to the message.

2.4. Single-Sideband Modulation

SSB transmits only one of the two sidebands of a DSB-SC signal. Using the Hilbert-transform representation, the upper-sideband signal may be written as

$$s_{USB}(t) = (A_c/2)[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)]$$

where $\hat{m}(t)$ denotes the Hilbert transform of the message. The ideal SSB bandwidth is

$$BW_{SSB} = W$$

SSB provides the highest bandwidth efficiency among the linear modulation methods considered here, although generation and coherent reception are more demanding than conventional AM.

2.5. Vestigial-Sideband Modulation

VSB transmits one complete sideband and a controlled residual portion of the opposite sideband. If f_v denotes the vestigial bandwidth, then

$$BW_{VSB} = W + f_v$$

A coherent VSB receiver can recover the message without amplitude distortion when the translated filter responses satisfy the additive-symmetry condition

$$H(f_c+f) + H(f_c-f) = \text{constant}, \quad |f| \leq W$$

2.6. Frequency and Phase Modulation

FM and PM preserve a constant carrier amplitude and therefore differ fundamentally from linear amplitude modulation. For a single-tone message $m(t)=\cos(2\pi f_m t)$, the FM and PM signals used in this experiment are

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_{FM} \sin(2\pi f_m t)]$$

$$s_{PM}(t) = A_c \cos[2\pi f_c t + \beta_{PM} \cos(2\pi f_m t)]$$

For single-tone FM, the peak frequency deviation is $\Delta f=\beta_{FM} f_m$. Carson's rule estimates the practical FM bandwidth as

$$BW_{FM} \approx 2(\Delta f + f_m) = 2f_m(\beta_{FM} + 1)$$

For the single-tone PM waveform above, β_{PM} is the peak phase deviation. The peak instantaneous-frequency deviation is $\beta_{PM} f_m$, so the same practical angle-modulation bandwidth form gives

$$BW_{PM} \approx 2f_m(\beta_{PM} + 1)$$

The theoretical spectra of FM and PM contain multiple sideband pairs at $f_c \pm n f_m$. Their practical bandwidths are therefore usually greater than the bandwidths of linear amplitude-modulation methods.

2.7. Comparison Criteria

The methods will be compared according to the following criteria:

- Carrier presence or suppression.
- Number and position of sidebands.
- Theoretical transmission bandwidth.
- Variation of amplitude, phase, or instantaneous frequency.
- Numerical mean power for the selected amplitudes.
- Envelope or coherent recovery requirement.
- Mean-square error between the original and recovered message for the linear modulation methods.

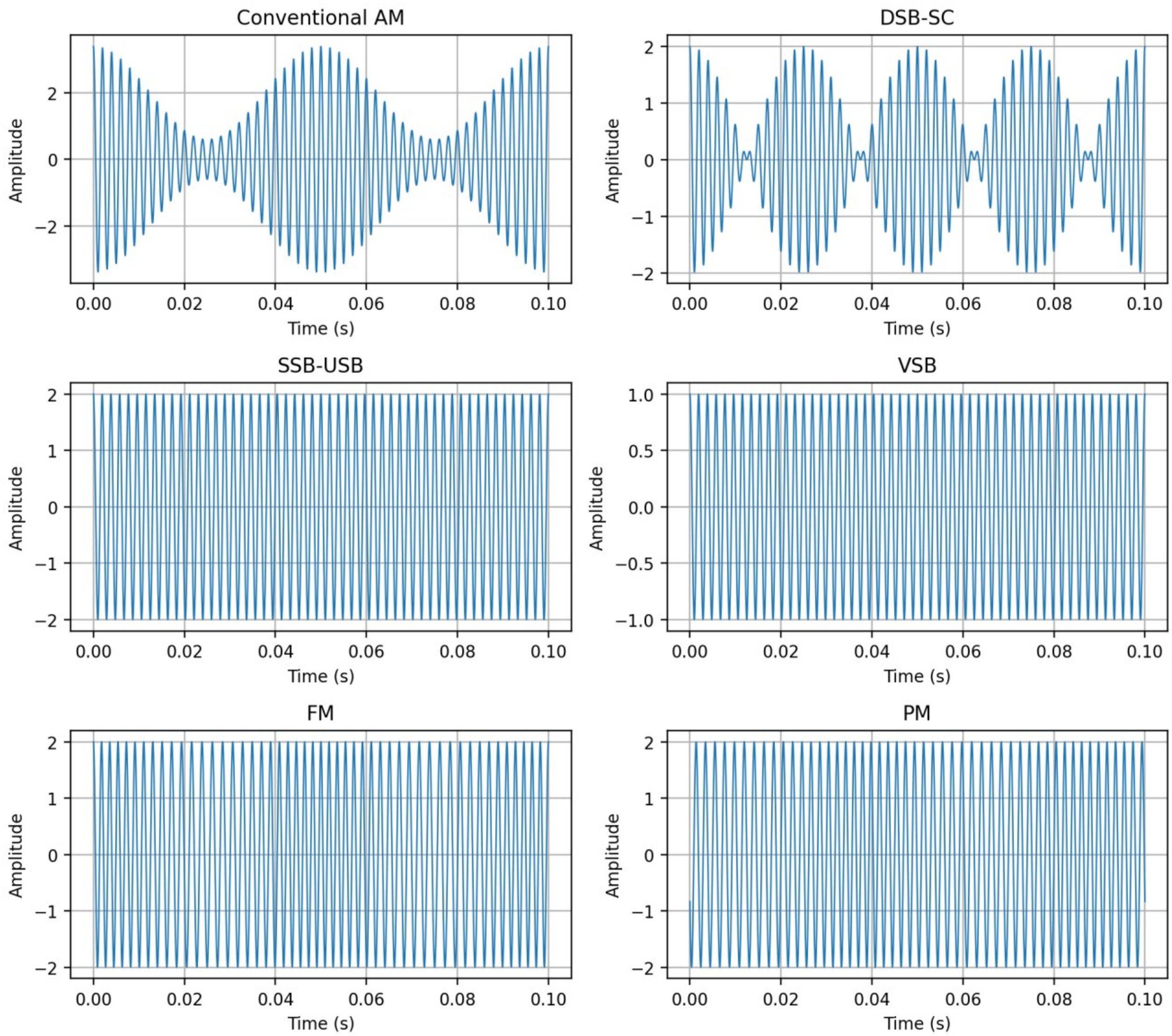

Fig. 10.2. Representative time-domain waveforms for the common experiment parameters.

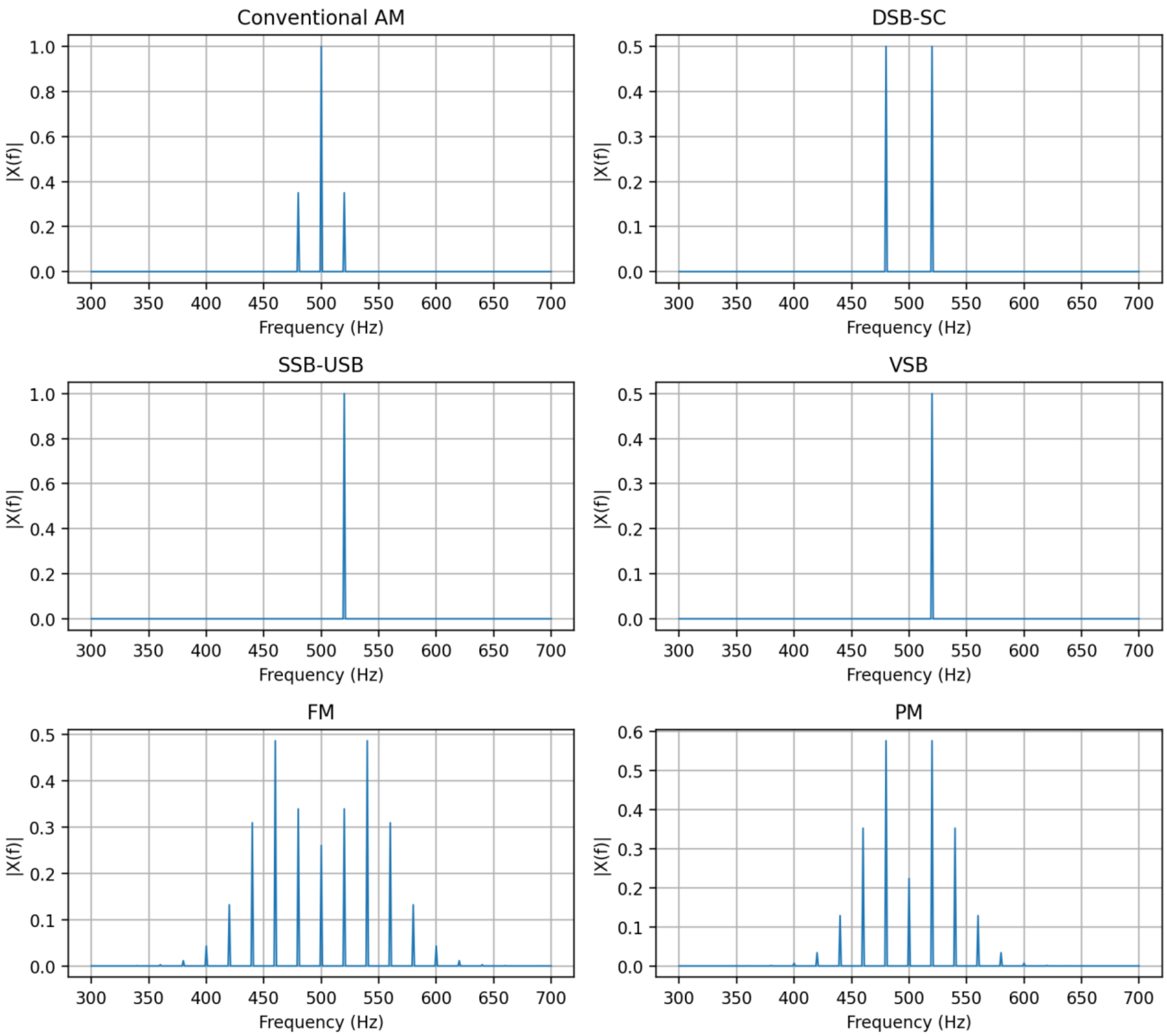

Fig. 10.3. Representative positive-frequency spectra for the modulation techniques.

3. Materials Used

- Computer
- MATLAB software
- MATLAB Script Editor
- MATLAB Signal Processing Toolbox or equivalent Hilbert-transform and filtering commands
- Introduction to Communication course notes
- Laboratory worksheet
- Calculator

The principal MATLAB commands used in this experiment are:

- cos, sin, pi, mean, max, abs
- fft, fftshift, ifft, ifftshift, fftfreq-equivalent vector construction
- hilbert, lowpass or an equivalent FFT-domain low-pass filter
- plot, stem, subplot, grid, legend, xlim, bar
- logical indexing and piecewise frequency-domain filter construction

4. Preliminary (PreLab)

Use the following reference parameters unless otherwise stated:

$m(t)=\cos(2\pi 20t)$, $A_c=2$, $f_c=500$ Hz, $\mu=0.7$, $\beta_{FM}=3$, $\beta_{PM}=2$, $f_v=8$ Hz, $F_s=20$ kHz

Complete the following tasks before attending the laboratory:

1. Write the mathematical expressions for AM, DSB-SC, USB, VSB, FM, and PM using the reference parameters.
2. List the positive-frequency carrier and sideband components expected for AM and DSB-SC.
3. State which single sideband is retained in the USB signal.
4. Calculate BW_{AM}, $BW_{DSB\text{-}SC}$, BW_{SSB}, and BW_{VSB}.
5. Calculate the FM peak frequency deviation and Carson bandwidth.
6. Calculate the PM peak instantaneous-frequency deviation and practical bandwidth estimate.
7. Rank the six modulation methods from the smallest to the largest theoretical bandwidth for the selected parameters.
8. Predict which methods contain a discrete carrier component at f_c.
9. State the appropriate recovery method for AM, DSB-SC, SSB, and VSB.
10. Explain why ideal FM and PM have constant envelopes.
11. Predict which methods are expected to have the highest numerical mean power when A_c is kept constant.
12. Complete Table 10.1 before the laboratory session.

Table 10.1. Preliminary comparison of the modulation techniques.

Technique	Carrier transmitted?	Theoretical bandwidth (Hz)	Expected recovery method
AM			
DSB-SC			
SSB-USB			
VSB			
FM			
PM			

5. Procedure

Part 1: Common Message and Carrier Signals

1. Open MATLAB and create a new script file.
2. Set $F_s=20$ kHz and define a one-second observation interval.
3. Generate $m(t)=\cos(2\pi 20t)$ and $c(t)=2\cos(2\pi 500t)$.
4. Plot the message and carrier in separate subplots.
5. Calculate the message bandwidth W and confirm the carrier frequency from the FFT.

```
Fs = 20000;
t = 0:1/Fs:1-1/Fs;
Ac = 2; fc = 500; fm = 20;
m = cos(2*pi*fm*t);
c = Ac*cos(2*pi*fc*t);
N = length(t);
f = (-N/2:N/2-1)*(Fs/N);
```

Part 2: Generation of the Linear Modulation Signals

1. Generate conventional AM using $\mu=0.7$.
2. Generate the DSB-SC signal by multiplying the message and carrier.

3. Generate an analytic representation of the message using the hilbert command and construct an upper-sideband SSB signal.
4. Generate a VSB signal by applying the complementary sideband-shaping response used in Experiment 8 to the DSB-SC spectrum.
5. Plot the four time-domain signals over the same short observation interval.
6. Calculate centered and normalized FFTs and identify the carrier and sidebands.

```
mu = 0.7;
sAM = Ac*(1 + mu*m).*cos(2*pi*fc*t);
sDSB = Ac*m.*cos(2*pi*fc*t);
ma = hilbert(m);
sUSB = Ac*real(ma.*exp(1j*2*pi*fc*t));
% Construct the VSB shaping response H(f) as in Experiment 8
% and calculate sVSB = real(ifft(fft(sDSB).*H));
```

Part 3: Generation of FM and PM Signals

1. Generate single-tone FM with $\beta_{FM}=3$.
2. Calculate $\Delta f=\beta_{FM} f_m$ and the instantaneous FM frequency.
3. Generate single-tone PM with $\beta_{PM}=2$.
4. Calculate the PM instantaneous frequency.
5. Plot FM and PM together with the message over a suitable time interval.
6. Confirm that both angle-modulated signals maintain constant amplitude.
7. Calculate and plot their FFT magnitude spectra.

```
betaFM = 3; betaPM = 2;
DeltaF = betaFM*fm;
sFM = Ac*cos(2*pi*fc*t + betaFM*sin(2*pi*fm*t));
fiFM = fc + DeltaF*cos(2*pi*fm*t);
sPM = Ac*cos(2*pi*fc*t + betaPM*cos(2*pi*fm*t));
fiPM = fc - betaPM*fm*sin(2*pi*fm*t);
```

Part 4: Bandwidth Comparison

1. Calculate the theoretical bandwidth of every modulation technique.
2. Create a MATLAB bar graph of the calculated bandwidths.
3. For each FFT, determine the principal occupied spectral interval using a common magnitude threshold.
4. Compare the measured intervals with the theoretical values.
5. Explain why the threshold-based FM and PM bandwidths may differ from Carson estimates.
6. Complete Tables 10.2 and 10.3.

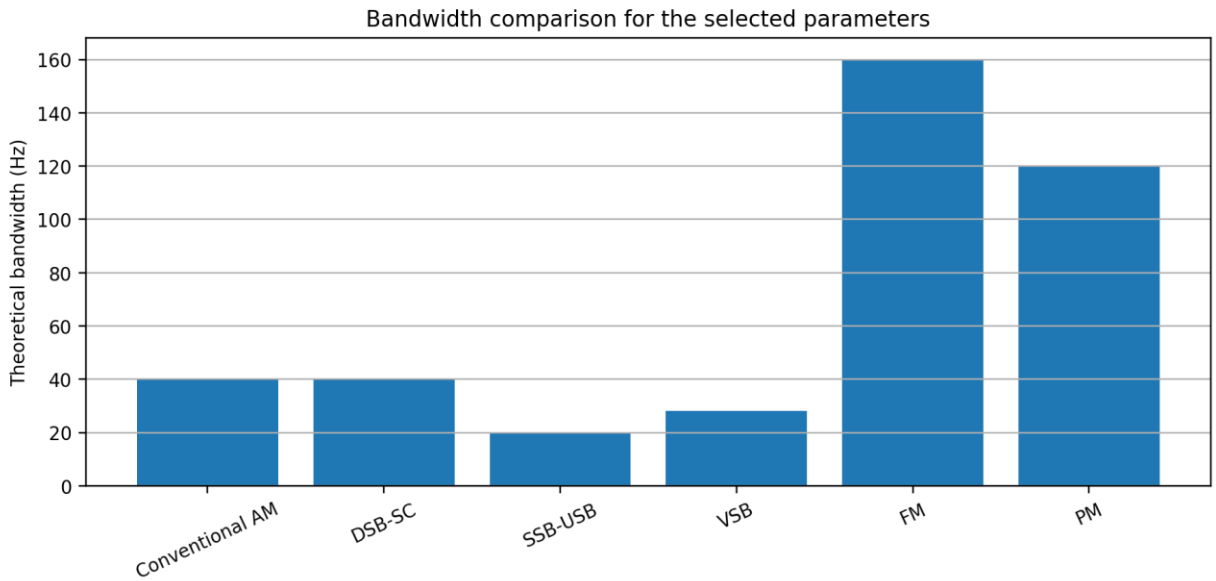

Fig. 10.4. Theoretical bandwidth comparison for the selected experiment parameters.

Part 5: Mean-Power Comparison

1. Calculate the numerical mean power of every signal using mean(abs(x).^2).
2. Plot the resulting powers using a bar graph.
3. Compare conventional AM with DSB-SC and explain the effect of the transmitted carrier.
4. Compare FM and PM with the linear modulation methods and relate their power to constant envelope operation.
5. Use identical Ac values throughout the comparison and state any scaling used for SSB or VSB.

```
P_AM  = mean(abs(sAM).^2);
P_DSB = mean(abs(sDSB).^2);
P_USB = mean(abs(sUSB).^2);
P_VSB = mean(abs(sVSB).^2);
P_FM  = mean(abs(sFM).^2);
P_PM  = mean(abs(sPM).^2);
```

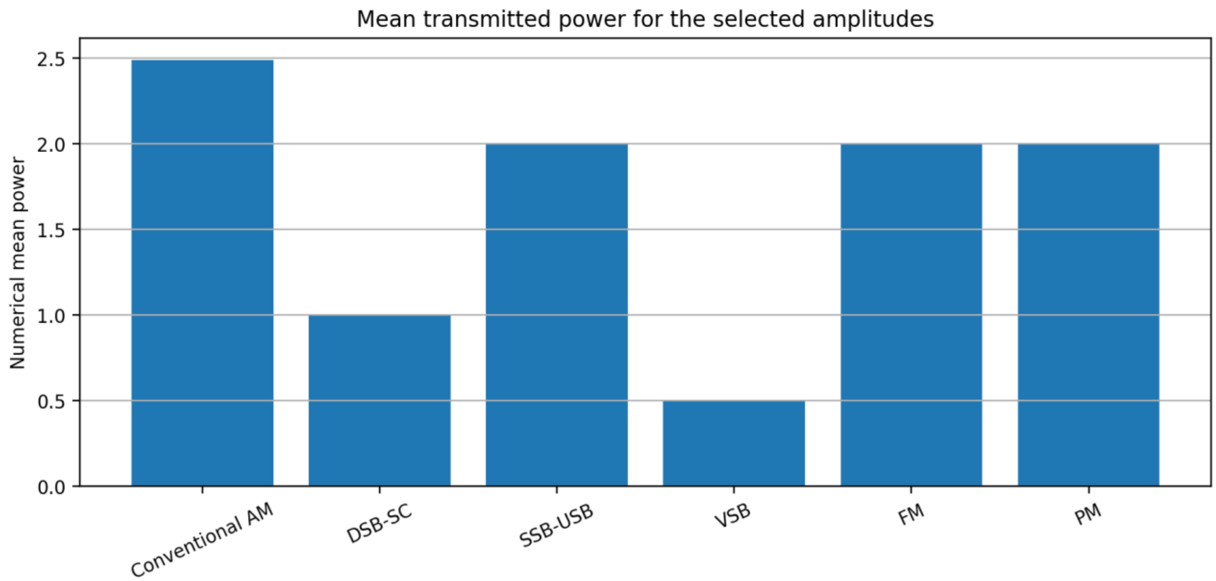

Fig. 10.5. Numerical mean-power comparison for the selected signal amplitudes.

Part 6: Recovery of the Linear Modulation Signals

1. Recover the AM message using the signal envelope obtained from the analytic signal.
2. Remove the DC component and divide by the modulation index and carrier amplitude.
3. Recover DSB-SC by multiplying the received signal by a synchronized cosine and applying a low-pass filter.
4. Recover USB and VSB using the same coherent carrier and a suitable low-pass filter.
5. Normalize the recovered amplitudes before waveform comparison.
6. Plot the original and recovered messages in the same figures.
7. Calculate the mean-square error and correlation coefficient for each recovered signal.
8. Do not apply envelope detection to DSB-SC, SSB, or general VSB signals; explain why it is unsuitable.

```
envAM = abs(hilbert(sAM));
mAM = lowpass((envAM/Ac - 1)/mu, 30, Fs);
lo = cos(2*pi*fc*t);
mDSB = lowpass(sDSB.*lo, 30, Fs);
mUSB = lowpass(sUSB.*lo, 30, Fs);
mVSB = lowpass(sVSB.*lo, 30, Fs);
% Normalize each recovered waveform before calculating MSE.
```

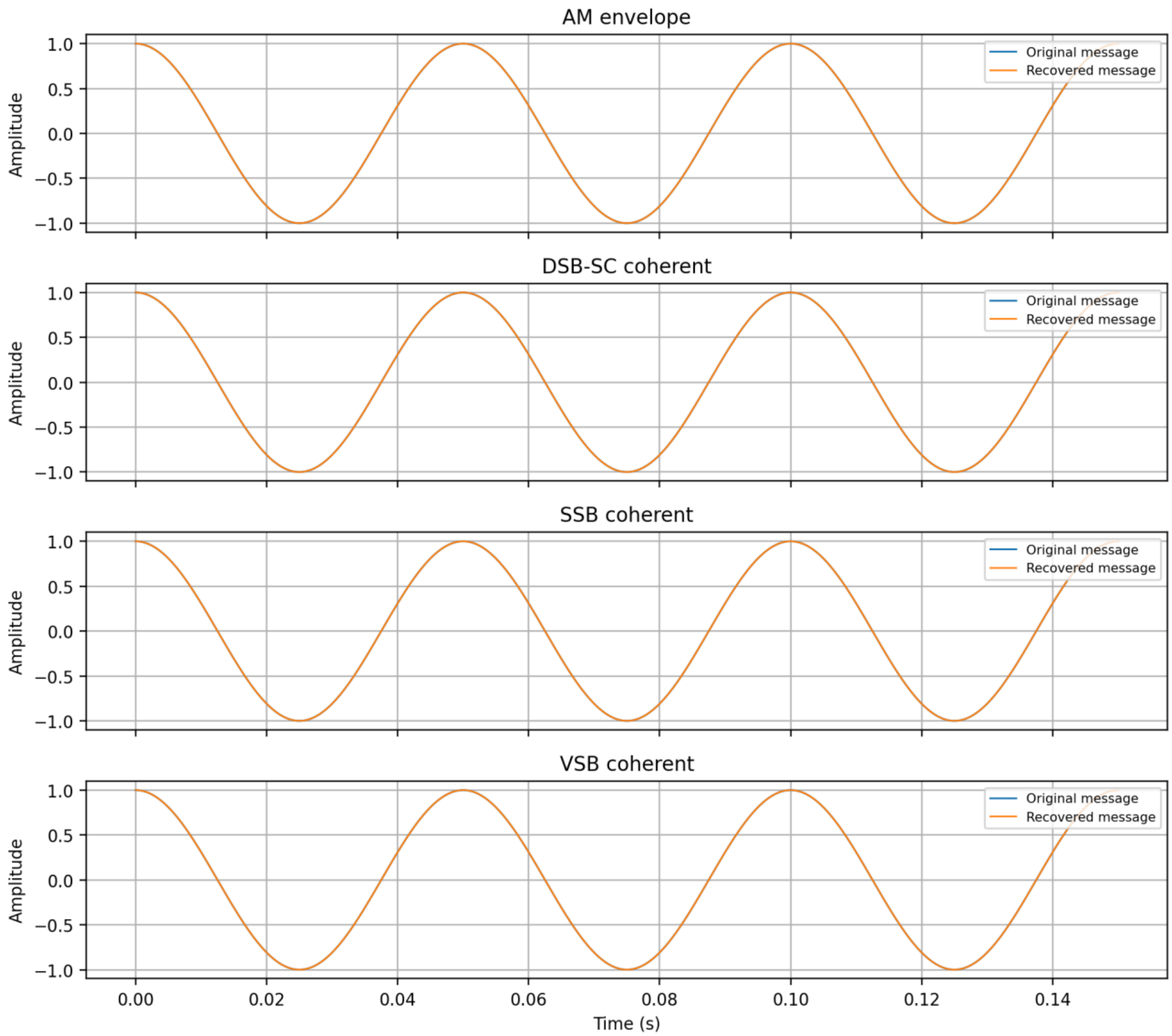

Fig. 10.6. Recovery of the message from the linear modulation signals.

Part 7: Final Comparative Interpretation

1. Prepare one final table containing carrier presence, bandwidth, mean power, recovery method, and measured recovery error.
2. Rank the modulation methods in terms of bandwidth efficiency.
3. Identify the methods that require coherent carrier synchronization.
4. Identify the methods with constant-envelope transmission.
5. Explain why no single modulation technique is optimal for every communication-system requirement.
6. Save the MATLAB script, figures, completed tables, and written interpretations according to the laboratory supervisor's naming convention.

6. Evaluation of Results

Tabulate and interpret all data obtained during the experiment.

Table 10.2. Measured spectral properties.

Technique	Carrier frequency (Hz)	Lower spectral edge (Hz)	Upper spectral edge (Hz)	Measured bandwidth (Hz)
AM				

Technique	Carrier frequency (Hz)	Lower spectral edge (Hz)	Upper spectral edge (Hz)	Measured bandwidth (Hz)
DSB-SC				
SSB-USB				
VSB				
FM				
PM				

Table 10.3. Theoretical and measured bandwidth comparison.

Technique	Theoretical bandwidth (Hz)	Measured bandwidth (Hz)	Percentage difference	Bandwidth rank
AM				
DSB-SC				
SSB-USB				
VSB				
FM				
PM				

Table 10.4. Power and waveform comparison.

Technique	Numerical mean power	Carrier transmitted?	Constant envelope?	Principal varied parameter
AM				
DSB-SC				
SSB-USB				
VSB				
FM				
PM				

Table 10.5. Message-recovery results.

Recovered signal	Detection method	MSE	Correlation coefficient	Observed distortion
AM				
DSB-SC				
SSB-USB				
VSB				

Table 10.6. Final qualitative comparison matrix.

Criterion	AM	DSB-SC	SSB	VSB	FM	PM
Bandwidth efficiency						
Power efficiency						
Receiver complexity						
Carrier synchronization requirement						
Envelope variation						
Primary advantage						
Primary limitation						

Answer the following questions:

1. Which modulation methods contain a discrete carrier component at fc?
2. Why do conventional AM and DSB-SC have the same nominal bandwidth but different power usage?
3. Why does SSB require only half the bandwidth of AM or DSB-SC?
4. How does VSB provide a compromise between SSB and DSB-SC?
5. Why is the additive-symmetry condition important for VSB coherent detection?
6. Which techniques require coherent carrier synchronization at the receiver?
7. Why can conventional AM be recovered using an envelope detector?
8. Why is an envelope detector unsuitable for DSB-SC and general SSB/VSB signals?
9. Which modulation methods maintain a constant envelope?
10. Why do FM and PM generally occupy more bandwidth than the linear modulation methods in this experiment?
11. How does increasing βFM affect the FM spectrum and Carson bandwidth?
12. How does increasing βPM affect the PM spectrum and peak phase deviation?
13. Why can a threshold-based numerical bandwidth differ from a theoretical bandwidth expression?
14. Compare the numerical mean powers and explain the effect of carrier transmission and scaling.
15. Which modulation technique is most bandwidth-efficient for the selected message?
16. Which technique provides the simplest noncoherent receiver?
17. Explain why the selection of a modulation technique involves trade-offs among bandwidth, power, and implementation complexity.
18. Summarize the principal MATLAB evidence obtained for each of the six modulation methods.